
BICYCLE IDLER DRIVETRAIN ANALYSIS
IN ASSOCIATION WITH NORCO BICYCLES

by

Denton Anderson

Jordan Donaldson

Kelly James

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Faculty Advisor: Stephen McMillan, M.Eng, P.Eng

Program Head: Mehrzad Tabatabaian, B.Eng, M.Eng, PhD, P.Eng

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Abstract

This report covers the research and testing of the rumbling phenomenon found in the drivetrains of high-pivot rear suspension mountain bikes, specifically those made by Norco Bicycles. This includes the project definition and objectives, a theoretical background of the problem, the development and testing of an analytical model, the design and development of a physical test bench, discussion of test results, the applicable findings, and the final conclusion.

The problem being addressed in this project is the drivetrain rumbling found in high-pivot mountain bikes. High-pivot mountain bikes allow for less momentum losses when rolling over square-edge bumps than a low-pivot bike, though an idler sprocket needs to be added near the pivot point to address the excessive chain growth. This idler is thought to be the cause of this rumbling; it is the objective of this project to research this phenomenon and try to discover a drivetrain design that minimizes the effect.

It was hypothesized that the polygonal nature of the idler, through the pitch polygonal effect, is the main cause for the rumbling. Due to the idler being a polygon, the radius varies as the chain engages and disengages with the sprocket, thus causing slight changes in the gear ratio. These changes can be felt by the rider as a rumbling. By changing the tooth count and position, therefore the wrap angle of the idler, the changes in gear ratio at both ends can be brought either into phase or out of phase by 180° . The project team hypothesized that the ideal setup would be at one of these two extreme cases.

To discover the optimal design, the testing was broken up into two groups: first, simulations were run through an analytical model, and second, the theoretical results were validated through testing on a physical test bench.

MATLAB was used to create the analytical model. The model was a 2-dimensional representation of the chainring, idler, and cassette, each made up of discrete points.

Different parameters such as tooth count, and relative position could be specified. It was chosen that the simulations would be performed at 25% bike sag, the position riders would be pedaling. The final output of the simulations was maximum change in gear ratio for a specific tooth count, as well as optimal relative position for the idler for a specific tooth count. It was found that the best case, given a number of assumptions, was using a 14-tooth sprocket, and the worst case was using an 11-tooth socket. It was decided that these tooth counts, as well as the Norco standard 16-tooth and second best 18-tooth would be experimentally tested.

To allow for data validation, a test bench was designed and manufactured. It was designed to transmit a constant force from a hanging weight through the drivetrain to a scale on the other end, where the tension could be read. The relative positions of both the cassette and the idler could be adjusted, to allow for various sag positions and wrap angles. Through additional pulleys at the weight and the scale, the reduction would be amplified to allow the scale to read the changes in chain tension. The pulley sizes were chosen by calculating the change in chain tension for a set weight and comparing that to the scale's resolution.

To discover an optimal solution, various variables were tested. These include idler tooth count, idler position, sprocket material, tooth profile, and the chain-line. Five individual

tests were run for each to allow for more consistent results. The results of each were compared to see how they influence the drivetrain performance.

After testing was completed, that both the tooth count and the idler position had the greatest effect on the change in gear ratio. For tooth count, a 14-tooth idler resulted in significant reductions in gear ratio change compared to an 11-tooth idler, almost a 94% reduction. By moving a 16-tooth idler to its theoretically optimal position, reductions in gear ratio change of 38% were observed. However, it was found that both the tooth profile of the idler, as well as its material made little difference.

Through this, it was found that a possible optimal bike frame design could exist, and through more thorough research using the above methods, drivetrain rumbling could be reduced to negligible levels during the design of a high pivot mountain bike.

The project was completed as of May 5, 2019. The project will be showcased at the BCIT Engineering Expo on May 10, 2019.

Acknowledgments

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1 – Introduction

In this section of the report, the background of the problem will be discussed, detailing the need for high pivot bicycles, along with their inherent problems. Additionally, the objectives of this project will be covered in order to justify the capstone project.

1.1 – Project Background

Generally, mountain bikes use a low-pivot suspension system that results in the rear wheel of the bike moving toward the center of the bike during compression as shown in Figure 1.2.

The advantage of this design is that there is minimal chain growth, allowing for a generic bicycle drivetrain to be utilized. The downside of this is that during square edge impacts the force is not tangential to the wheel motion path, and as a result, the rider experiences a backwards component of the force and is slowed down while riding.



Figure 1.1 High Pivot Suspension [1]



Figure 1.2 Traditional Low Pivot Suspension [2]

High-pivot suspension systems have become a popular option on many mountain bikes in recent years due to their performance advantages over low-pivot suspension systems. The main advantage of a high-pivot suspension system is that under compression, the rear wheel path is away from the center of the bike as shown in Figure 1.1. This means under square edge hits the rear wheel is allowed to absorb the bump vertically and horizontally. This allows the rider to continue moving forward with minimal momentum losses. The downside, however, is that there is significant chain growth during compression, which causes the crank arms to rotate with compression of the suspension. This significantly disorients the rider and can be quite dangerous.

The common solution to this problem is to add an idler sprocket to the drivetrain of the bike, placed so that pitch radius passes through the center of the main suspension pivot as shown in Figure 1.3. This results in the upper section of the chain, between the chainring and the cassette, experiencing little to no growth, while the growth in the lower section, between the derailleur and the chainring, is taken up by the derailleur. However, this idler does lead to efficiency losses and a “rumbling” sensation felt while pedaling in some situations.

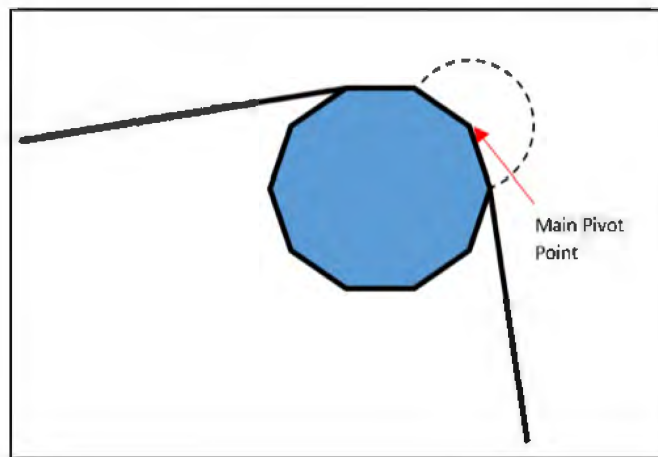


Figure 1.3 Depiction of Idler Position

Due to these inefficiencies, the majority of high-pivot mountain bikes manufactured today have been downhill bikes, which are highly gravity assisted and do not require the rider to pedal for long periods of time. However, some manufacturers are now looking at the possibility of using a high-pivot suspension system in their enduro and trail bikes, which are

used for both descending and climbing. This makes it necessary to analyze the effects of the idler on the bicycle drivetrain and how its effects could possibly be negated.

1.2 – Project Objectives

To the best of the team’s knowledge prior to this project, no bicycle manufacturers have analyzed the effect which these idlers have on drivetrain efficiency or how to best negate the effects. The main objective of this project is to analyze different idler tooth counts, idler positions, and tooth profiles, and also investigate how each variable effects the performance of a bicycle drivetrain as well as on the “rumbling” sensation found on these high-pivot mountain bikes.

2 – Detailed Description of the Current Status

There are many possible explanations for the inefficiencies and the “rumbling” sensation created by the addition of the idler to the bicycle drivetrain. One possible point of interest is the pitch polygonal effect of chainrings creating a changing gear ratio. Other possibilities include the tooth profile as well as the chain-line, which could be responsible for the effects described by riders.

2.1 – The Pitch Polygonal Effect

The pitch polygonal effect refers to the inherent noncircular nature of a chainring. The radius is not consistent all the way around which results in the gear ratio changing slightly as the chainring is rotated. This polygonal shape is shown in Figure 2.1. This effect is amplified with smaller chainrings or sprockets as the angle between points is larger, causing a larger gear ratio change with each rotation between points on the polygon.

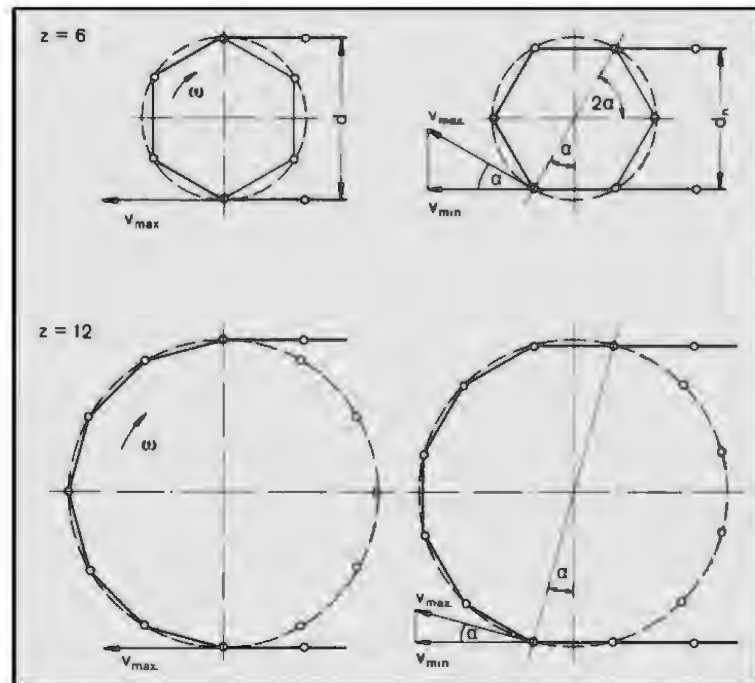


Figure 2.1 Pitch Polygonal Effect Depiction [3]

2.2 – Idler Sprocket Problems

With the introduction of the idler sprocket into the drivetrain, there are a number of problems which arise due to the polygonal nature of the sprocket. One problem being the changing force on the chain as the sprocket as is rotated. The equation for torque is:

$$T = F_T * r$$

Looking at Figure 2.2, if the input force at the pedals stayed constant the torque produced by Tension 1 would be:

$$T1 = (Tension\ 1) * (Radius\ 1)$$

Where Radius 1 is the radius at the point the chain comes onto the idler sprocket with the accompanying Tension 1.

As the chain passes over the idler sprocket, it will come off at a certain angle specified by the bicycle geometry and position in travel. Due to this, the exiting radius could be anywhere within the bounds of the polygon. As such, based on the same equations, with zero motion:

$$\Sigma T = 0, \quad T_1 + T_2 = 0, \quad F_{T2} = \frac{r_1}{r_2} F_{T1}$$



Figure 2.2 Free Body Diagram of Idler Sprocket

As such, these alternating radii can be negated by moving or changing the size of the idler and/or other components to change the wrap angle on the idler. By changing the wrap-angle

the chain enters and leaves the idler at different relative angular points. This means that you can alter the rate at which the radii change, and consequently the tensions. There are essentially infinite scenarios in this relationship with two extremes; with the in and out radii completely in phase, or 180° out of phase. The design team believes the optimal case of this relationship will be directly at one of these two extremes. These two cases represent the range of all options for this system.

2.3 – Alternative Causes to Be Investigated

An alternative cause to be investigated is the chain-line of the bike. This is how close to in-plane the selected rear cassette sprocket, the front idler sprocket, and chainring are in relation to each other. When these components aren't in plane, the chain bends, which creates friction and inefficiencies in the drivetrain. On most bicycles the chain-line is optimized in the middle of the cassette to give a good compromise between the longest and shortest gears on the cassette. However, with many downhill bikes, the chain-line is optimized for the bottom half, or the longest gears, of the cassette because that is what is used most often. It was noted that for the Norco Aurum HSP, the chain bent a significant amount when in the lowest “climbing gear”.

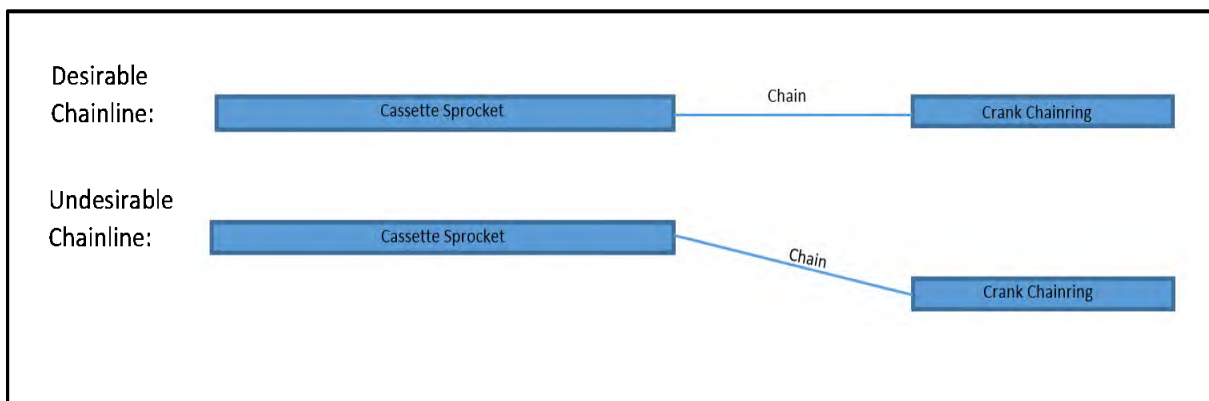


Figure 2.3 Depiction of Chain-line Characteristics

Another possible cause could be the tooth profile of the idler. If the chain does not engage properly with the teeth of the idler sprocket, the changing tensions could cause the chain to shift back and forth tangentially, which might explain the “rumbling” feeling. In the physical

tests, different tooth profiles will be compared with the same tooth count and their results compared. There are generally two teeth profiles used by most manufacturers for idler sprockets: standard tooth, and a narrow wide tooth profile, shown in Figure 2.4. The narrow wide tooth profile gives better engagement with the bicycle chain as it compensates for the varying in chain internal spacing with each link and could possibly influence any movement brought on by the changing tensions.



Figure 2.4 Narrow Wide Tooth Profile

3 – Theoretical Background

In this section of the report, the reasoning behind the design team's methods will be explained. Specifically, the approach with which the mathematical system was created, and how the test bench will be used to validate the simulation results.

3.1 – Numerical Simulation Approach

In order to accurately simulate the high-pivot mountain bike suspension, a fully numerical simulation approach was chosen, utilizing MathWorks MATLAB which would represent the two-dimensional geometric system. The components of the system that needed to be represented include the front chainring, the selected cassette gear, and the idler sprocket.

Each of the sprockets require two key aspects: an accurately placed center point, around which the sprocket can rotate, and an array of points to represent each tooth. These points are required to be spaced one-half inch apart (12.7 mm) to represent the real sprocket on a bicycle and are required to be spaced at the pitch diameter governed by Equation 3.1 below.

$$D = \frac{P}{\sin\left(\frac{180}{N}\right)}$$

Equation 3.1 [4]

Where:

D = Pitch diameter

P = Chain Pitch

N = Number of Teeth

With this set of three sprockets placed on a two-dimensional plane, the entire system could then be rotated as the real-world bicycle drivetrain does. In this way, the design team could ensure the model maintained the desired geometry and could watch the simulation run to verify it was working properly.

As the simulation was being done in MATLAB, the design team planned to take advantage of the computational power of the software by utilizing iterative mathematics to simplify the complex relationships that governed this system. This would allow for the calculation of parameters using simple trigonometry and loops, rather than complex equations which would have to be derived, costing the team large amounts of time.

3.1.1 – Simulation Assumptions

As with any mathematical model, certain assumptions had to be made. These assumptions were made with the goal of simplifying calculation parameters by a reasonable amount. This allowed for the design team to focus the tests to the probable causes of this phenomenon, while also allowing for reasonable computation times. The assumptions of the simulation are as follows:

- The pitch of the chain is exactly one-half inch (12.7mm), ignoring effects of any stretching from worn chains.
 - The chain line exists on a flat two-dimensional plane. Any effect due to variation from this can be tested using the physical test bench.
 - The effects of rider ‘bob’ experienced in pedal strokes does not contribute to this effect, therefore simulations can be run in a static position of suspension travel.
 - This effect scales linearly with rider input force due to the geometric nature.
 - The pivot location of the bicycle suspension exists approximately at the 45-degree mark of the idler sprocket along the chain center line.
 - Sprocket tooth profiles cannot be tested utilizing this simulation; therefore, any effects due to this will be tested using the physical test bench.
-

3.2 – Data Validation Methods

One key aspect to this project is the validation of any results obtained in the simulation portion of this project. As such, the design team attempted to ensure the validity of the simulation results with a real-world test bench, which both represents the real-world bicycle drivetrain, and isolates the same variables as the mathematical model. This would remove any uncertainty about mathematical artifacts appearing as results in the simulations.

If time constraints do not limit the project substantially each analytically tested combination of sprocket tooth counts, and positions will be validated using the test bench. This would allow for a reasonable degree of certainty with regards to the accuracy of the mathematical model.

It should also be noted that to ensure sound scientific practice, all physical tests will be done repeatedly to prevent any unintended measurement artifacts or errors from skewing the data.

4 – Description of the Project Activity and Equipment

In this section of the report, the project activity will be described in detail. This includes descriptions of both the simulation, and the test bench portion of the project. This description will include the development of the specific MATLAB code, as well as the development of the test bench.

4.1 – Development of MATLAB Scripts

The ability for the design team to accurately, and with repeatability, predict the effect of the idler sprocket on the system was the key result of this project. As such, much care was taken to ensure the MATLAB scripts were developed in a manner which ensured the desired assumptions were met. Additionally, the desired input values could be easily modified, and the desired outputs were achieved.

4.1.1 - Simulation Workflow

In order to develop an efficient and understandable simulation, a workflow was developed in advance to the development of the individual scripts. The workflow is as follows:

First, the input variables are entered into the simulation; these are covered in Section 4.1.2 of this report. From here, the simulation is broken into main simulation scripts, and sub-function scripts to perform individual sets of calculations. These scripts are available with brief descriptions in Appendix A.1. For the purposes of clarity, this section will be broken up by these scripts.

MoveIdler:

With the current Aurum HSP specifications, the bicycle main pivot location passes through the chain line of the 16 tooth idler sprocket at approximately the 45-degree angle from the positive x axis. After talking with David Cox at Norco Bicycles, we determined it was crucial to maintain this relation between the chain path and the pivot location. As such, this sub-function was developed. The function inputs the desired tooth count for any test and moves

the center location of the idler to maintain the chain line passing through the 45-degree radial position of any sprocket.

Create_Circle:

Next, the radial tooth positions of all three sprockets are created on the two-dimensional plane. This is done by running the Create_Circle function three times. This function creates three two-dimensional arrays, with a column of x positions, and a column of accompanying y positions. These arrays are all centered on the zero position when created, with a radius governed by pitch diameter relationship seen in Equation 3.1. After this, the points of the idler and cassette arrays are shifted by adding the center location to the values of all the points in the array.

Init_Position_KI & Init_Position_IC:

Now the system needed to be rotated in order to accept the virtual one-half inch pitch chain. In order to do this, the Init_Position functions were used for the crank to idler section of chain, and the idler to the cassette section of chain respectively. The functions work by incrementing the rotation of the idler array, and the cassette array respectively, this is done in small increments of 0.001 degrees. After each increment, the length between the active teeth was checked with respect to a function of one-half inch; if the length was not function of one-half of an inch, another increment was done, if it was, the initial position had been found, and the function would close.

At this point, the system was set up, and a loop was created to increment by the desired amount, until the desired angle of crank rotation was reached. The following functions exist inside this loop and are done at each measured position.

FindTangent_KI & FindTangent_IC:

In order to determine the tooth at which the chain leaves any given sprocket, the team utilized a method of finding the furthest away tangent line, therefore showing the engaged tooth on each sprocket. These functions determined two active teeth each; between the crank and idler, and the idler and cassette. This was done by drawing a line between all combinations of points on the desired sprockets and then equating these lines. After this, the

function would determine the upper most line, and the right most line between the idler and cassette, and the idler and crank respectively.

SpinRatio:

The last step to determine the movement of the drivetrain system was to determine the active gear ratios between any two sprockets. This was done by evaluating the current apparent radii for the crank sprocket, the crank side of the idler sprocket, the cassette side of the idler sprocket, and the cassette sprocket. These radii were found by sweeping through a reasonable range of angles through the sprocket and determining the minimum value, therefore exposing the perpendicular apparent radius as shown in Figure 4.1. These four radii created three distinct gear ratios; the ratio of the crank to the idler, the ratio of the idler to the cassette, and as a result, the net ratio from the crank to the cassette. Due to the polygonal nature of the sprockets, this apparent radius would change therefore varying these ratios.

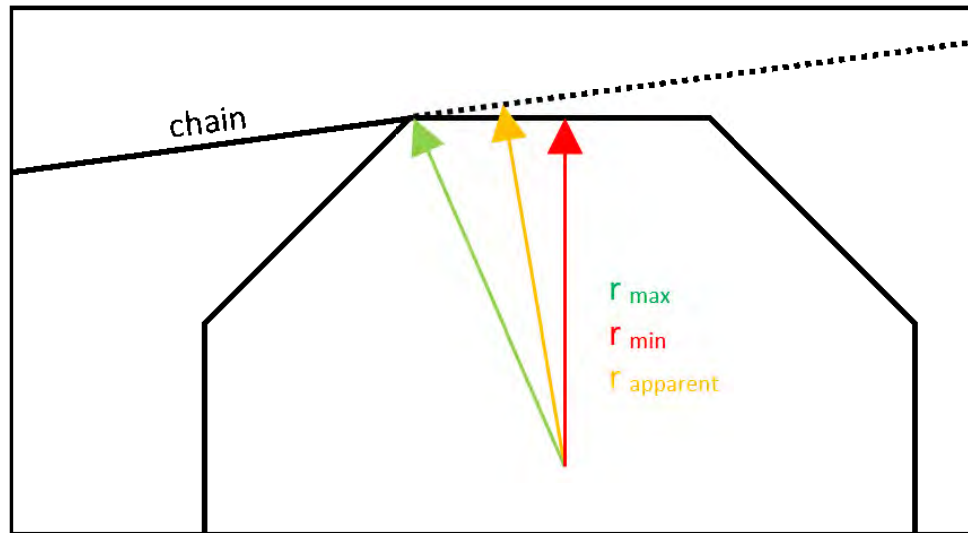


Figure 4.1 Apparent Radius Depiction

4.1.2 – Input Variables to Simulation

Firstly, the input variables to the simulation were required to be determined. These were selected to be the following:

- Idler center position (x and y coordinates)

- Cassette center position (x and y coordinates)
- Front chainring tooth count
- Idler sprocket tooth count
- Cassette sprocket tooth count
- Total angle of crank rotation to simulate
- Discrete change in angle per measurement point

With these basic geometric input variables, the design team felt confident that any possible scenario could be tested with regards to the real-world bicycle.

From here, two more complex, simulations would require additional input variables to allow for both a range of positions to be tested along with a range of sprocket tooth counts.

Therefore, the additional input values were required for these tests:

- Distance to allow x and y movement of idler in positive and negative x and y axis
- Discrete step size for x and y movement
- Range of idler tooth counts to simulate

With these additional input variables, both of these range tests could be simulated respectively.

4.1.3 – Treatment of Results

With these scripts running in a loop, and with the addition of some simple trigonometry, the simulation is able to accurately simulate a rotating drivetrain system for a single test. For a single test run, the output results contained a single plot detailing the overall gear ratio value throughout the rotation of the system. This plot allowed the design team to view the expected profile of the gear ratio variation through the rotation of the system.

From here, the design team required that multiple tests be run to efficiently compare multiple cases. For the purposes of these tests, the design team decided the best method to compare many tests was the comparison of the maximum change in gear ratio over a single simulation. This value would be the amplitude of the gear ratio variation, and therefore, the value desired to be reduced. In order to create these test runs, secondary and tertiary loops

were utilized, repeating the system until the desired tooth count, or position variation was completed, and the results were stored.

For the creation of plots, a combination of MATLAB and Microsoft Excel was utilized. Any two-dimensional plots were created using MATLAB to streamline the process and to create high quality plots. For the position variation tests, the team wanted a way to both see a heat map of the high and low values produced, while also being able to quickly and easily read these values. For this, conditional formatting was used in Excel to create a simple, yet effective heat map, with the values located directly in each cell. These heat maps can be seen in Appendix B.2.

4.1.4 – Interpretation of Results

After completion of all simulations, due to the sheer number of possible tests to be done, the design team decided to ensure there were three key comparisons to test. First, a set of reasonable tooth count profiles (over a set rotation) would be saved and compared to the upcoming test bench results to test for similarity. Second, a set of tooth count tests would be performed at the sag position to find the optimal, and worst-case tooth counts; in the event of validity shown by the first results, these could be used as a guide for future bicycle design given a required idler position. Finally, the same method would be done for the position variation tests, allowing for design use in the case of matched profiles to the real-world tests once again to be used for development of fixed tooth count bicycles in the case of validity.

4.2 – Development of Test Bench

The test bench was designed to be modular and allow for extremely precise adjustment to allow for different bicycle drivetrain components in a variety of orientations. The design also had to be able to consistently measure miniscule changes in drivetrain forces and slowly record them in very small increments in order to mimic the pseudo-static nature of the simulation.

A design session was conducted, and a number of possible test bench designs were analyzed. Aluminum extrusion was chosen to make up the main structural members of the test bench due to it being relatively stiff, light, and highly modular. A large portion of the bench would be made of bicycle drivetrain components with the remaining portion being made of pulleys connecting both to a scale, and a hanging mass. The hanging mass would replicate the rider input pedaling load on the drivetrain in a consistent and predictable manner. The scale would allow for measurement of small changes in the drivetrain forces and allow for easy data recording.

4.2.1 – Mechanical Design Process

To reduce costs, and for ease of manufacturing, aluminum extrusion was utilized from a previous capstone project and made up the skeleton of the test bench. The aluminum extrusion allows for ease of adjustment and allows the test bench to be as modular as possible. By connecting the aluminum extrusion members with t-nuts and bolts, the members can be moved fore and aft along tracks which allow for nearly infinite adjustment of the test bench.

For the drivetrain side of the test bench shown in Figure 4.2, off-the-shelf bicycle components were chosen. They allowed the test bench to accurately replicate real world forces seen on a mountain bike and simplify manufacturing because they merely need to be mounted to the frame to function. A SRAM GX 12 speed drivetrain was chosen because it represents the newest in bicycle drivetrains and offers a large climbing gear which is necessary to replicate a drive mode (climbing) in which this “rumbling” is critical.

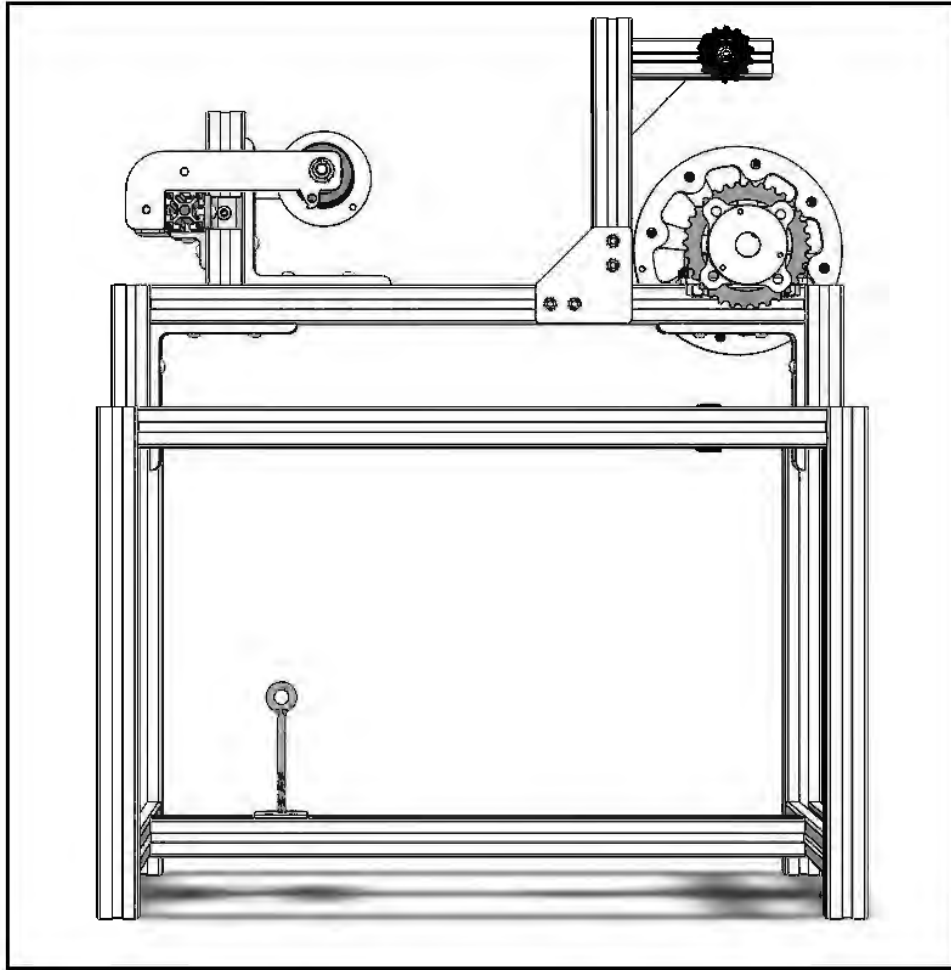


Figure 4.2 Chain Side Test Bench

A 148mm mountain bike hub was chosen to accurately represent the modern enduro bike and to simplify manufacturing. With the mountain bike hub, the cassette could be mounted as it would be on a bike and the rear pulley could be mounted by the disc rotor mounting bolts.

On the other side of the test bed shown in Figure 4.3, a hanging weight would represent a constant force exerted on the pedals of a bike by the rider. A gym weight was chosen due to its relatively accurate weight and ease of procurement. It was initially hung and connected to an aluminum pulley by nylon climbing rope, but after running the first test it was determined that the rope was stretching, so the team switched to steel cable.

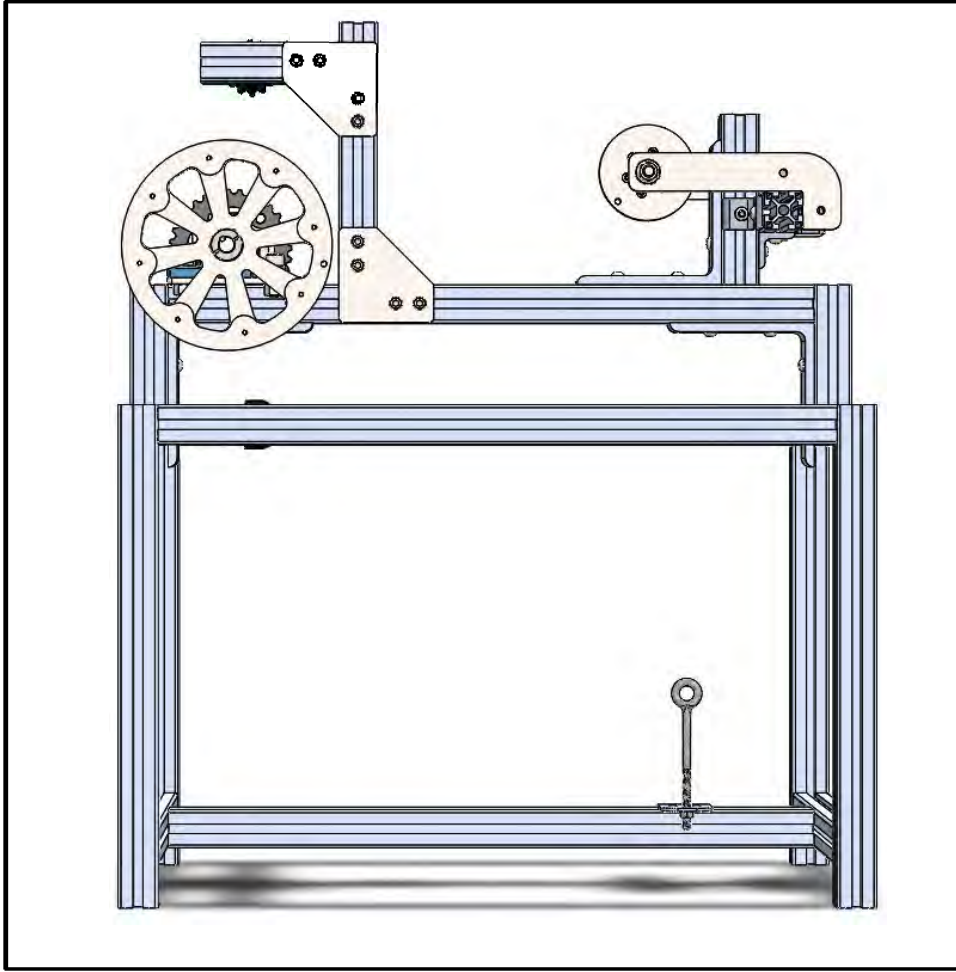


Figure 4.3 Pulley Side Test Bench

The front pulley was mounted on a $\frac{3}{4}$ " axle with a $\frac{3}{16}$ " keyway and a shaft collar to eliminate side to side movement. The $\frac{3}{16}$ " keyway was mathematically determined to be a suitable size and would easily cope with the torques placed on it by the test bench. The axle itself was mounted to two bearing blocks procured from a previous capstone project to lower costs and simplify designing. This assembly is shown in Figure 4.4.

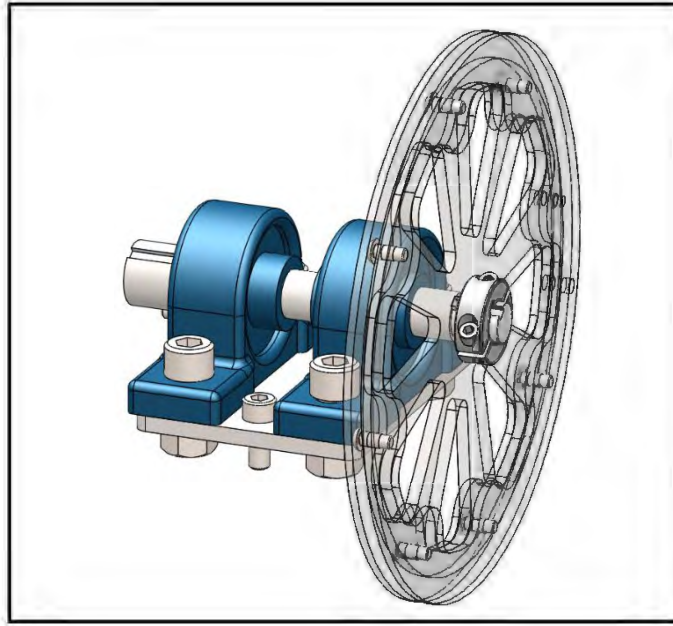


Figure 4.4 Crank Axle Assembly

The chosen measuring device was a Performance Tool 660lb digital scale. It was given to the team by the team advisor Stephen McMillan and, again, reduced costs and simplified the design. The scale's force rating and accuracy was mathematically determined to perform acceptably in the required tests as shown in the next section 4.2.2 Pulley and Weight Sizing.

The system would be cycled by turning a nut on an eyelet, shown in Figure 4.5, affixed to the scale. As the scale was let out, the cassette would turn, which would turn the chainring and idler, and then turn the front pulley which would lower the weight. This would allow the system to be turned slowly and allow for miniscule changes to be recorded to fully capture any gear changes present in the system, no matter how small.

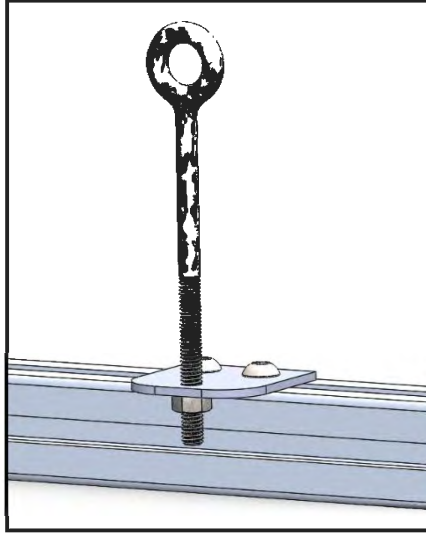


Figure 4.5 Slow Release Eyelet Assembly

If there is any confusion on the location of parts in the test bench please reference Appendix C which contains clearly labeled technical drawings of the test bench.

4.2.2 - Pulley and Weight Sizing

To determine the size of both the front and rear pulleys, both the weight of the test bench plate and the resolution of the scale had to be considered. The radius of the pulleys had to be such that they created enough reduction to allow the scale to be able to read the change in chain tension as the overall gear ratio changed from its maximum to its minimum. If this change was too small, the scale would not show a change in chain tension. Additionally, the pulleys were required to fit within spatial constraints of the test bench.

A MATLAB program called Load_Cell_Calc was written to calculate this change in tension, for both the worst-case scenario and the best-case scenario; refer to Appendix A.2 for the specific script. Based on a set radius for both the front and rear pulleys, as well as the maximum and minimum and minimum overall gear ratios (obtained by the system simulation), the program would calculate the change in scale tension for set range of masses (kg). The output of the program was a list of masses applied to the system and the corresponding changes in scale tension.

Comparing the program's output with the scale's resolution, 0.1 kg, a minimum mass could be chosen. Various pulley radii were run through the program until a reasonable minimum

mass was found; around 25 lbm. As can be seen in Table 4.1, an example output of the program, the highlighted row has the lowest mass with a change in tension that can be read by the scale, in this case, an 18 kg mass with a change in tension of 0.206 kg.

This method, however, did not take into account a number of variables that could affect the scale's reading. For example, frictional losses, frame bending, and chain and cable elongation were all ignored. To combat this, a slightly more massive than necessary weight would be chosen. It should also be noted that the larger the weight used, the better the accuracy on the readings as this effect was speculated to scale linearly.

After a number of iterations, a front pulley radius of 100 mm, a rear pulley radius of 40 mm, and a mass of 25 lb. was chosen.

Table 4.1 Pulley Selection Results

Mass (kg)	Δ Mass (kg)	Mass (lb.)
23	0.2632	50.7150
22	0.2518	48.5100
21	0.2404	46.3050
20	0.2289	44.1000
19	0.2175	41.8950
18	0.2060	39.6900
17	0.1946	37.4850
16	0.1831	35.2800
15	0.1717	33.0750
14	0.1602	30.8700
13	0.1488	28.6650

4.2.3 – Manufacturing Methods

Many parts on the test bench were able to be ordered from online suppliers, which eliminated manufacturing time and ensured the team would meet project deadlines. However, some parts required manufacturing due to their custom nature. In order to produce the parts such that the design team was able to quickly and easily finish the test bench within the limited time frame, traditional manufacturing was minimized, and numerically controlled manufacturing methods were used wherever possible. As such, the majority of parts were manufactured using waterjet cutting methods, and additive manufacturing methods.

Waterjet cutting was used to create the gusset plates for the frame, the pulleys, bearing block mount, chainring mount, and the bicycle hub brackets. These parts can be seen below in Figure 4.6.

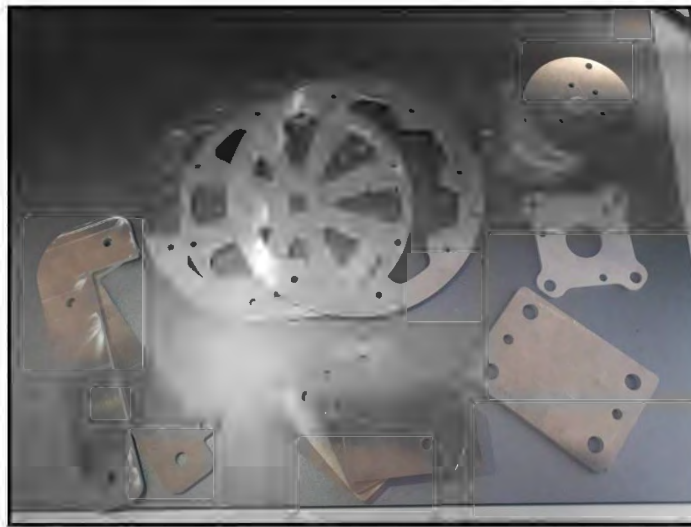


Figure 4.6 Waterjet Cut Parts

Additive manufacturing was used to create feet for the frame, extrusion end caps, the bicycle shifter mount, an indicator for more easily measuring of the horizontal location of the idler pulley, and the majority of the idler sprockets used in testing. These parts can be seen below in Figure 4.7 and Figure 4.8.



Figure 4.7 3D Printed Sprockets (left) and Shifter Mount (right)



Figure 4.8 3D Printed Center Marker (left) and Extrusion End Caps (right)

Finally, two components were manufactured using the manual lathe and mill: the crank drive axle, and a spacer for the idler sprocket mount. These parts were simplified for manual machining and as such only resulted in approximately 3 hours of manufacturing time. These parts can be seen below in Figure 4.9 and Figure 4.10. Additionally, the part drawings used for part manufacture can be seen in Appendix C.



Figure 4.9 Turned Idler Spacer

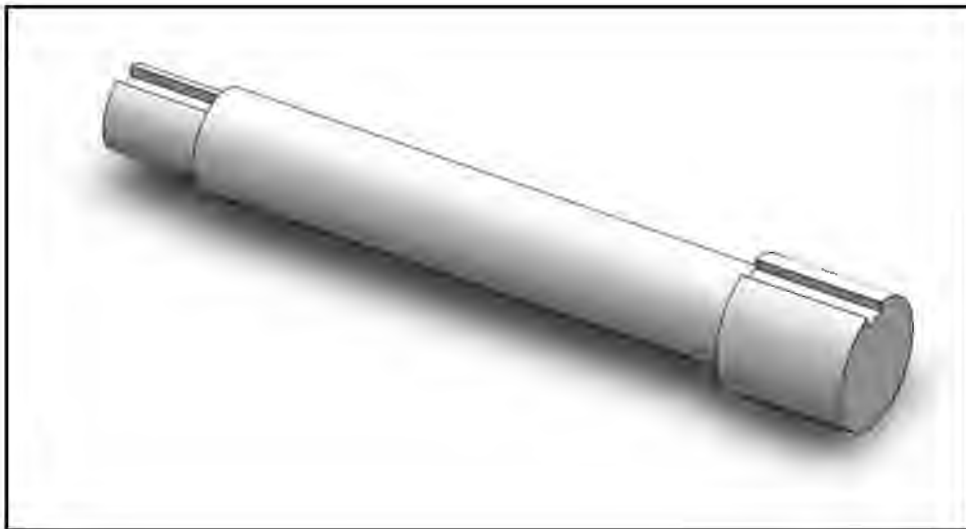


Figure 4.10 Manufactured Crank Axle

4.3.2 – Assembly and Positioning Methods

The aluminum extrusion members and their gussets were assembled using t-nuts and bolts. This allowed for fast assembly, simple adjustment, and fine tuning of the location of parts when necessary.

The pulleys, chainring adapter, chainring, bearing blocks, and mounting plate were sandwiched together with nuts and bolts for ease of assembly as shown in Figure 4.11. Also, 3D printed parts were designed to be clipped onto extrusions and other parts.

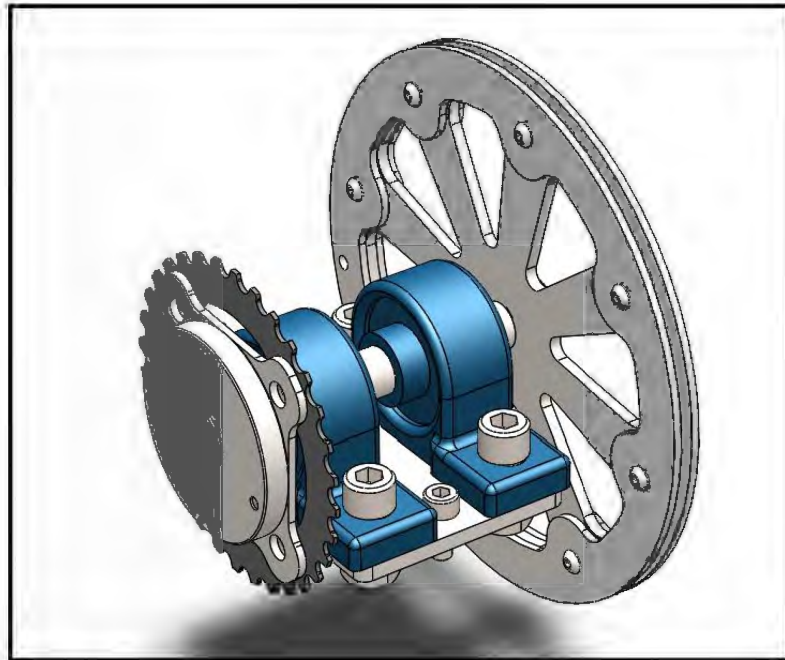


Figure 4.11 Crank Assembly

The positioning method utilized the SolidWorks model to calculate different positions for various bikes and idler sprocket sizes. All measurements are based off of the bottom bracket, or the $\frac{3}{4}$ " axle, using it as a zero point. From there, measurements are taken from the SolidWorks model and aluminum extrusions are moved on the test bench appropriately. To ensure four datum locations were used as known locations for measurements for the horizontal, and vertical locations of both the idler, and cassette locations, a set of digital calipers were used for measurement to ensure accuracy.

5 – Discussion of Results

In this section of the report, the results and findings of both the analytical simulations as well as the physical tests on the test bench will be discussed. The different factors of the tests, including idler tooth count, position of crank, idler, and cassette, chain line, and tooth profile will be covered on how they affected the final results. Also, problems and difficulties that had to be addressed and overcome by the project team will be discussed. Finally, the applicable findings that will be passed along to Norco will be covered.

5.1 – Project Findings

In this section of the report, the overall findings of the project will be discussed; this includes simulation results, test bench results, and comparison of the two. Additionally, data regarding alternative influences such as tooth profile, and chain-line will be discussed.

5.1.1 – Idler Tooth Count

The first variable that the project team tested was the tooth count of the idler sprocket. All of the tooth count tests performed on the test bench were done at 25% suspension sag which would represent the most realistic riding conditions. The bike represented in these tests is a 170mm travel enduro bike with similar geometry to a Norco Aurum HSP. All of the gear ratio results obtained from MATLAB are only the change in gear ratio seen in the drivetrain, this does not include the changes created by the pulleys on the test bench.

Based on the simulation results from MATLAB (Figure 5.1), it was found that the tooth count that would result in the smallest change in gear ratio would be 14 with a max change in gear ratio of roughly 0.0025, while the worst would be 11 with a max change of 0.0475. To verify the best- and worst-case results, it was decided that both an 11-tooth and a 14-tooth idler would be tested on the test bench. The second-best case, an 18-tooth idler, and, due to their use on the current Norco high-pivot suspension bikes, a 16-tooth idler will also be tested.

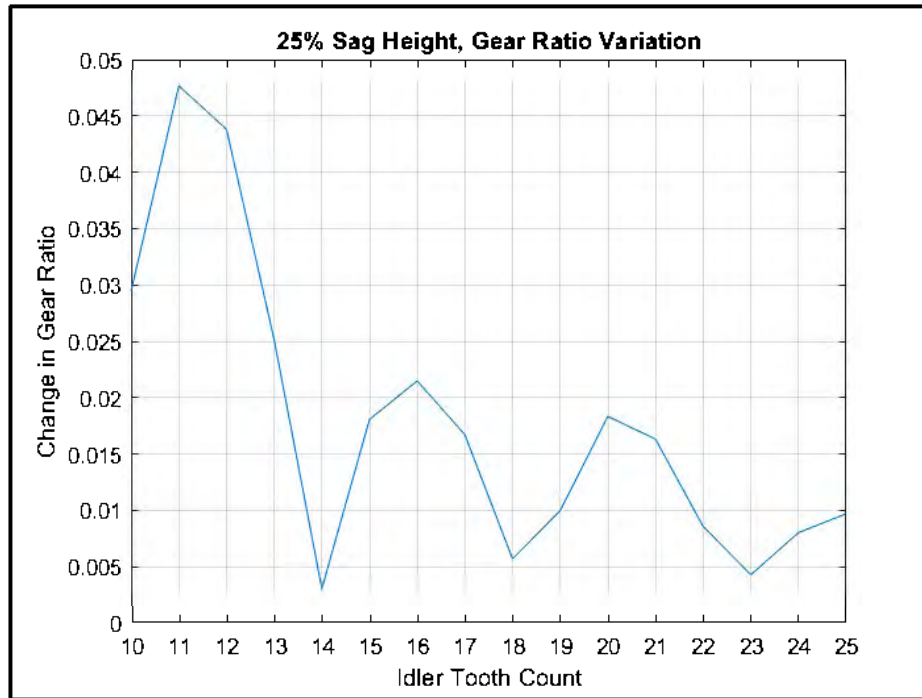


Figure 5.1 25% Sag Gear Ratio Variation Plot

5.1.1.1 – 11-Tooth Idler Sprocket

The first set of tests performed on the test bench were with the 11 tooth idler sprocket. This sprocket had a non-narrow-wide tooth profile and was made from plastic. Based on the MATLAB results, which can be seen as the blue line in Figure 5.2, the test should have shown the gear ratio changing from roughly 0.63 to 0.68, with a mean gear ratio of around 0.655. The change in gear ratio would also be expected to be in a saw tooth pattern.

The mean test results, represented by the green line in Figure 5.2, do seem to validate the MATLAB results for the 11-tooth idler. Both the change in gear ratio as well as the shape of function both roughly match the theoretical results. From this, it can be concluded that an 11-tooth idler is indeed non-ideal for a high pivot bike, resulting in a large amount of rumbling in the chain.

It should be of note however, that there is a significant offset between the theoretical MATLAB data and the test bench data. This issue could have a number of causes and will be discussed in further detail in a later section.

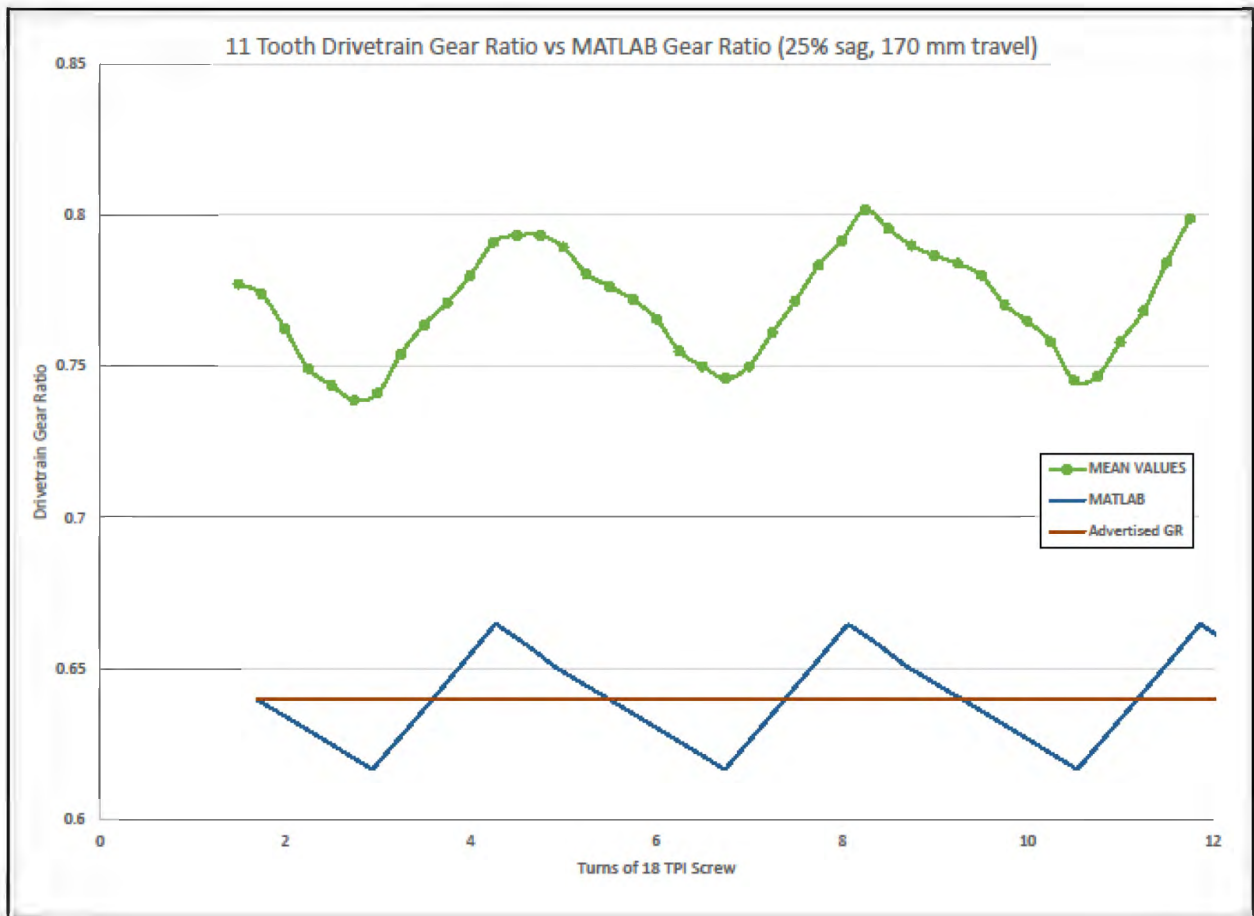


Figure 5.2 11 Tooth Idler Analysis Plot

5.1.1.2 – 14-Tooth Idler Sprocket

The second set of tests done was with a 14-tooth idler sprocket. The sprocket was made from plastic and had a narrow-wide tooth profile. This test was supposed to represent the best-case scenario found from the analytical simulations. As can be seen in Figure 5.3, represented by the blue line, the gear ratio from the MATLAB results should vary from roughly 0.638 to 0.641, with a mean gear ratio of 0.64. Based on this, using a 14-tooth sprocket should result in a 94% reduction in change in gear ratio as compared to the 11-tooth scenario.

The mean results of obtained from the test bench tests, represented by the green line in Figure 5.3, while showing a large reduction in gear ratio change, do not match the theoretical results quite as well as with the previous test. It is hypothesized that this issue is due to

limitations in the resolution of the scale used to measure the chain tension. Due to the relatively low weight of the mass used, the slight changes in tension could not be properly detected by the scale. A larger weight was used in another experiment to test this theory, which will be discussed in a later section. The differences are also likely due to friction and other losses in the system.

Despite these differences, the results obtained for the 14-tooth idler test, with a max change in gear ratio of roughly 0.0179, validate that 14-tooth idler would significantly reduce the rumbling felt in the drivetrain.

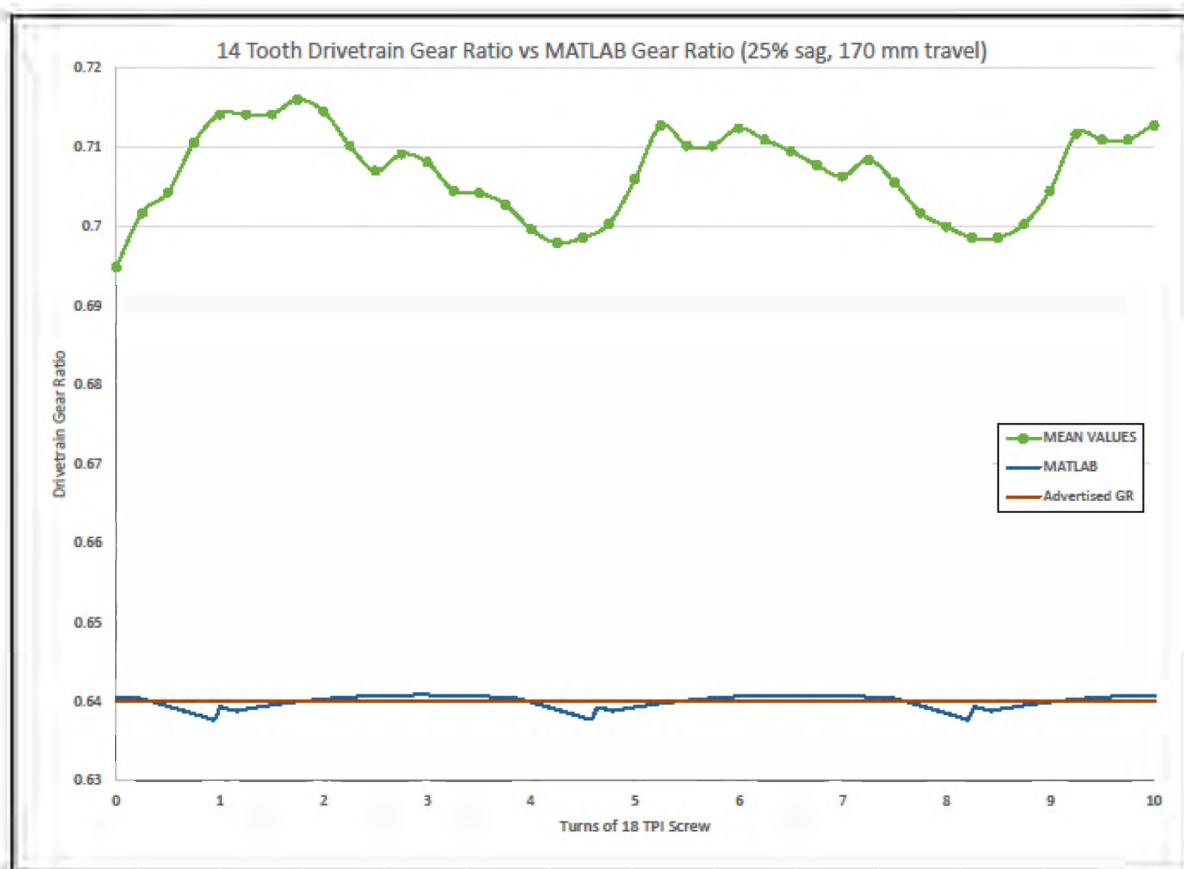


Figure 5.3 14 Tooth Idler Analysis Plot

5.1.1.3 – 16-Tooth Idler Sprocket

For the third set of tests, a 16-tooth idler sprocket, standard for Norco bikes, was used. This sprocket was made from aluminum and had a non-narrow-wide tooth profile. This was done to analyze the suitability of a 16-tooth idler, and how one compares to other tooth counts.

Based on the theoretical results obtained from MATLAB, represented by the blue line in Figure 5.4, using a 16-tooth idler should result in a moderate drop in change of drivetrain gear ratio as compared to an 11-tooth idler, though not getting as low as a 14-tooth idler. With a maximum change in gear ratio of roughly 0.63 to 0.65 (0.02) and a mean of 0.64, a 16-tooth idler results in a 60% reduction from an 11-tooth idler.

The mean results from the tests done on the test bench, represented by the green line in Figure 5.4, do seem to match the theoretical results quite closely, similar to the results from the 11-tooth tests. The max change in the gear ratio was found to be roughly 0.03, which while slightly lower than the theoretical change, is still significantly better than the results found during the 11-tooth idler test. Similar to the previous tests, the constant offset of the experimental data from the theoretical data is present.

From these results, it can be concluded that, while reducing the gear ratio change from the worst case, the 16-tooth idler can be improved upon by changing the tooth count. It is also of note that due to the fact that a 16-tooth idler produces more rumbling in the chain than a 14-tooth idler, the wrap angle of the chain over the idler is not the only factor in determining the amount of rumbling in the system.

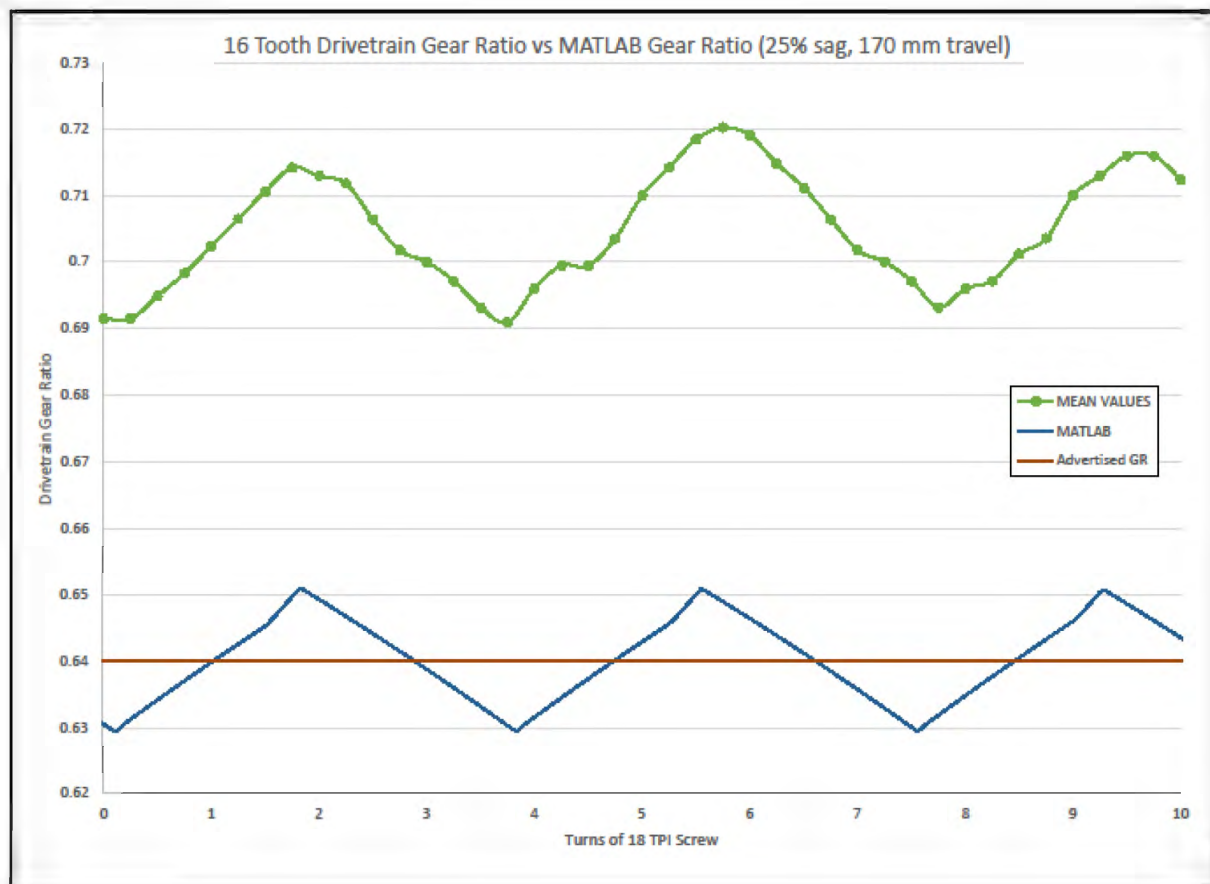


Figure 5.4 16 Tooth Idler Analysis Plot

5.1.1.4 – 18-Tooth Idler Sprocket

The final tooth count tested on the test bench was 18. This sprocket was made from plastic and had a narrow-wide tooth profile. This set up was tested due to an 18-tooth sprocket resulting in the second-best reduction in gear ratio change.

Based on the theoretical results obtained from MATLAB, represented by the blue line in Figure 5.5, using an 18-tooth sprocket should result in relatively low changes in gear ratio. The maximum expected change in gear ratio would be from approximately 0.6431 to 0.6374 (0.0057), with a mean gear ratio of 0.64. This is an 88.6% reduction in gear ratio change from the worst-case 11-tooth set up, though it is roughly 47.4% worse than the best-case 14-tooth idler.

The experimental mean obtained from the tests on the test bench, represented by the green line in Figure 5.5, does seem to validate the theoretical results. The maximum change in gear ratio, 0.6753 to 0.6667 (0.0086), is quite close to the maximum change found in the theoretical results, with the difference due to a number of factors including friction losses. It should be of note that, similar to the 14-tooth tests, the scale resolution posed an issue. The minute changes in chain tension were likely too small for the scale to detect.

Overall, based on the results gathered, an 18-tooth idler sprocket would result in smooth drivetrain operation, significantly smoother than an 11-tooth or 16-tooth idler. However, due to the large size of the sprocket, and the slightly better results from the 14-tooth sprocket, it would likely be better to use a 14-tooth idler.

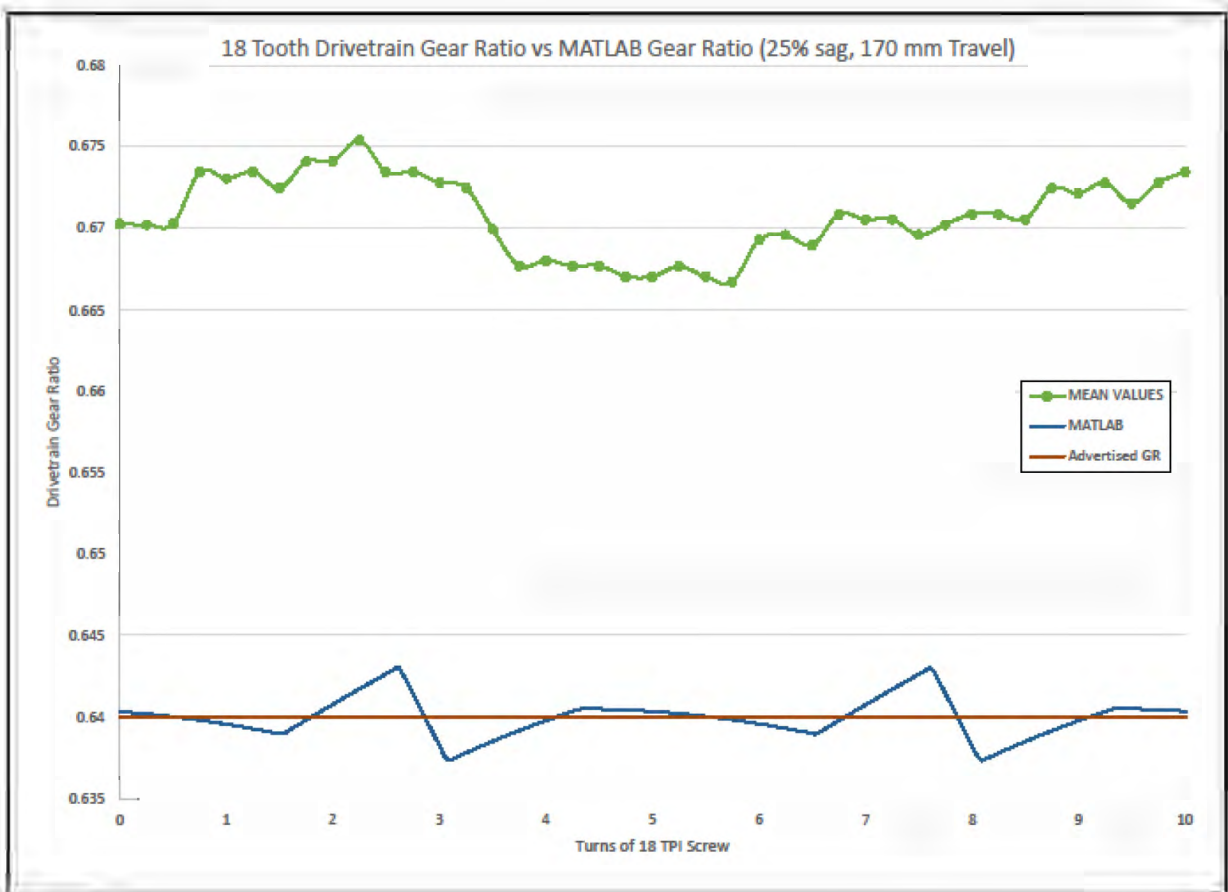


Figure 5.5 18 Tooth Idler Analysis Plot

5.1.2 – Idler Position

A second variable that was thought to affect the change in the overall gear ratio was the position of the idler. For all four tests involving the tooth counts, the idlers were positioned such that their pitch radiuses always lined up at the point 45° from the horizontal. It was found through MATLAB, however, that these positions were not always the most optimal. For example, a 16-tooth idler was theoretically supposed to result in smaller changes in gear ratio if moved up and to the right by roughly half an inch. This is the setup that was tested.

The theoretical results obtained from MATLAB for this new position, represented by the red line in Figure 5.6, indicate that there should be a max change in gear ratio of approximately 0.0124. This would be a 38% decrease in max change in gear ratio of the idler in its original position, down from 0.02, as obtained from the purple line in Figure 5.6.

The experimental results obtained from the tests in the new position, indicated by the green line in Figure 5.6, seem to validate the theoretical results. It can be seen that the optimal position, with the max change in gear ratio varying between 0.7225 and 0.7028 (0.0197), has only a slightly worse change than the theoretical results. Further, the experimental results for the original position, which was discussed in section 5.1.1.3 and is represented by the blue line in Figure 5.6, has a max change in gear ratio of roughly 0.03, thus validating the differences in the two positions.

Through these results, it is evident that positioning an idler based on its tooth count has a significant impact on the change in overall gear ratio. It should of note, however, that the positioning of the idler is limited by the geometry of the bike frame.

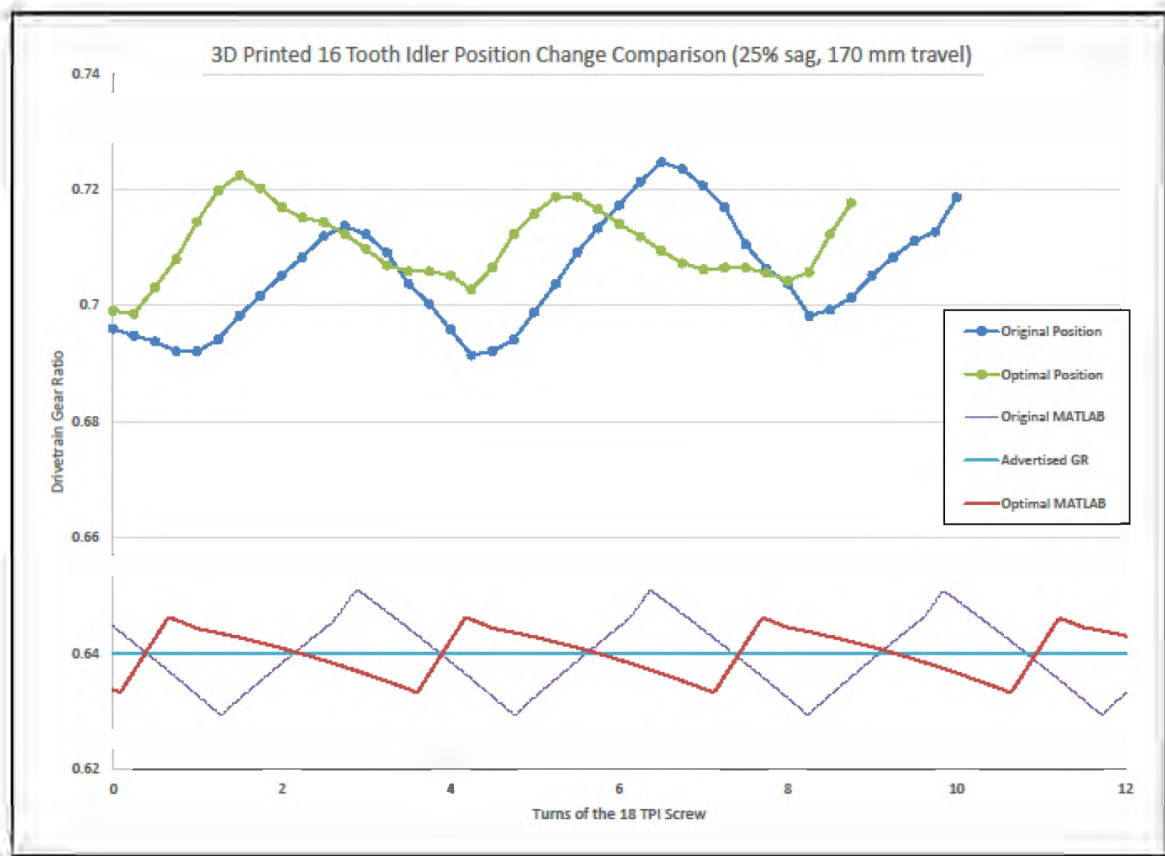


Figure 5.6 Idler Movement Analysis Plot

5.1.3 – Idler Tooth Profile

Another possible variable that could affect the change in the overall gear ratio would be the tooth profile of the idler. Two different aspects of the tooth profile were tested: the first is the length of the tooth, and the second is a narrow-wide vs non-narrow-wide profile.

5.1.3.1 – Long Tooth vs Short Tooth 11-Tooth Idler Sprocket

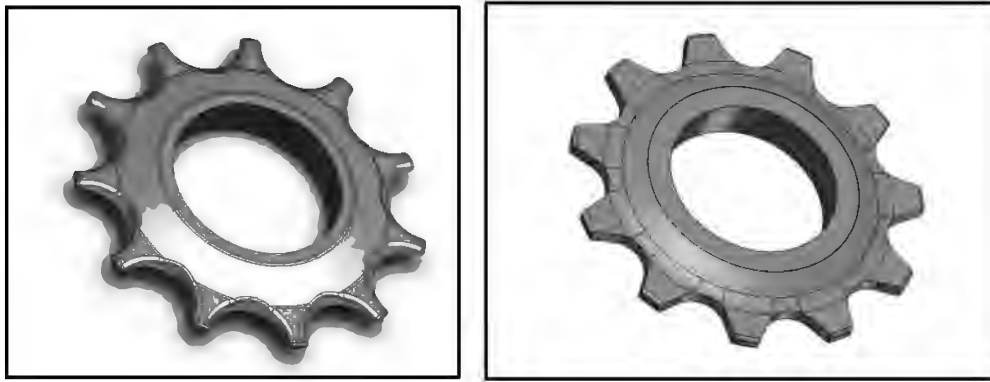


Figure 5.7 Long Tooth Sprocket Profile (Left) and Short Tooth (Right) 11 Tooth

To test how the length of the tooth would affect the change in gear ratio, two different 11-tooth idler sprockets were tested: the first was the original sprocket, which had a longer tooth profile (Figure 5.7), the second sprocket has a stubbier tooth profile (Figure 5.7). Both sprockets were made of PLA and had a non-narrow-wide profile. The tests were performed at 25% sag.

As can be seen in Figure 5.8, there is a significant difference in the results from the two idlers. The longer tooth idler, represented by the grey line, has similar max changes in gear ratio to the theoretical results from MATLAB. Refer to section 5.1.1.1 for more details on these results. The shorter tooth idler, represented by the green line, while having similar max changes in gear ratio as the long tooth idler, at 0.0524, has much smaller offset from the theoretical results.

The improved offset of the short tooth idler could be a result of a number of things. It is possible that the different bearing allowed for less frictional losses, or the use of a slightly different mass caused a change in the results.

Despite the difference in magnitude of gear ratio, the max change in gear ratio of the two idlers is almost the same. This would imply that the length of the teeth does not significantly impact the change in gear ratio.

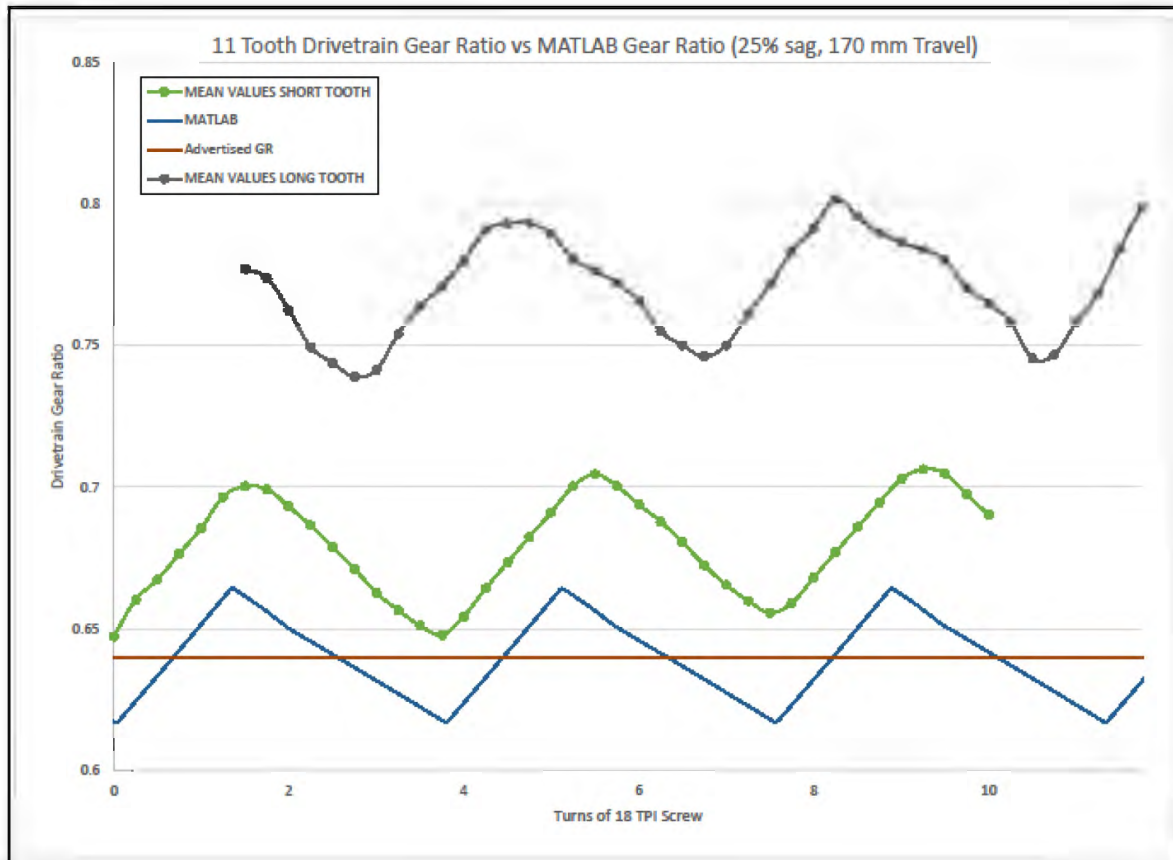


Figure 5.8 Tooth Length Idler Comparison Plot

5.1.3.2 – Narrow-Wide vs Non-Narrow-Wide 16-Tooth Idler Sprocket

To test if a narrow-wide tooth profile can affect the change in the overall gear ratio, two separate idlers were tested: one with a non-narrow-wide profile, and one with a narrow-wide profile. Both idlers were 16-tooth and were made from aluminum. Both tests were performed at 25% sag.

The mean of the results from the test with the first sprocket, the one with a non-narrow-wide profile, represented by the blue line in Figure 5.9, is discussed in a previous section. Refer to section 5.1.1.3 for further details. The mean results from the narrow-wide idler, represented by the green line in Figure 5.9, seem to be similar to those of the first idler with the exception of a smaller offset from the theoretical results from MATLAB. The max change in gear ratio

varies from 0.6618 to 0.6831, with a total change of 0.0213. This is only slightly worse than the non-narrow-wide change of 0.02.

Based on these results, it would seem that a narrow-wide tooth profile has little effect on the overall change in gear ratio. The difference in offset, similar to that in the other tooth profile test case, can be explained by the use of a different weight or using a smoother bearing.

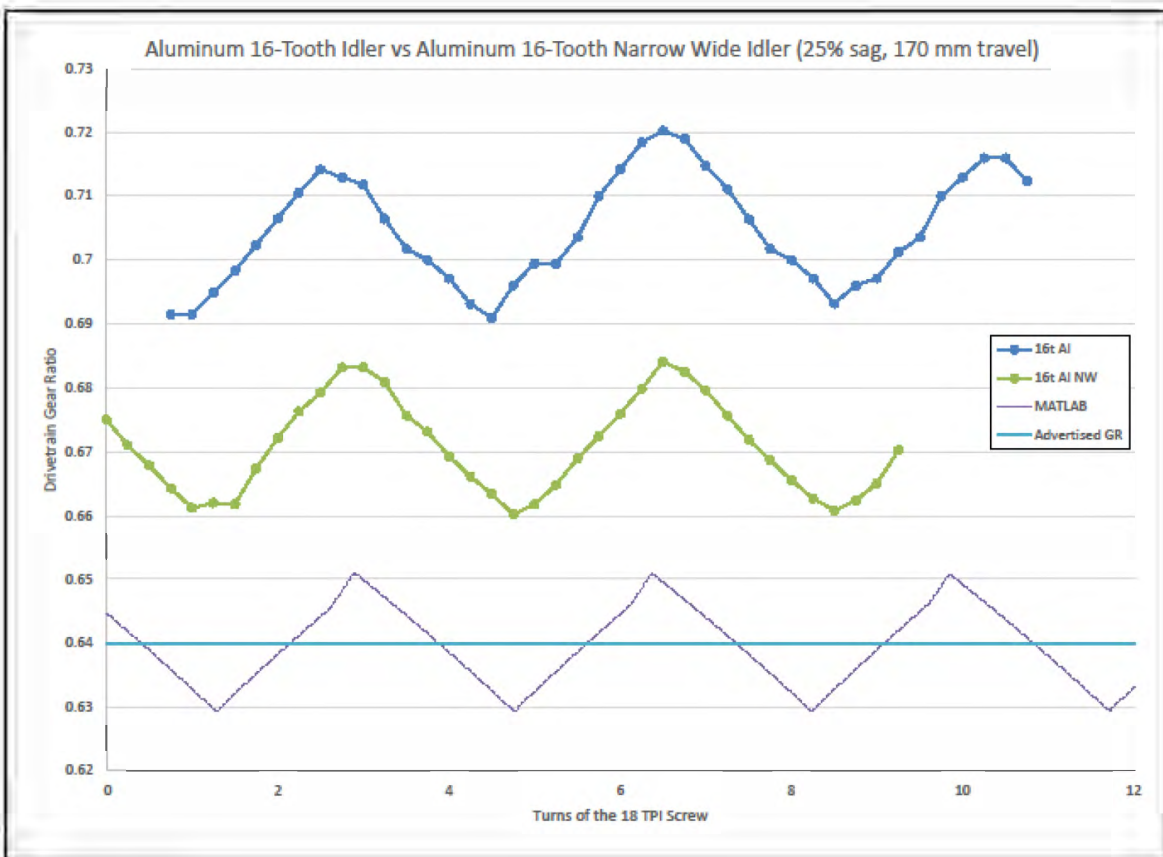


Figure 5.9 Narrow Wide Idler Comparison Plot

5.1.4 – Idler Sprocket Material

Another variable tested was the idler sprocket material. It was thought that it was possible that the different stiffness's of each material could affect the behavior of the chain as it passed over the idler sprocket. Two 16-tooth sprockets were tested and then compared: the first was made from aluminum and had a non-narrow-wide tooth profile, the second was

made from 3D Carbon PLA and had a narrow-wide tooth profile. Both tests were performed at 25% sag.

To see how the different materials compared, the mean results from each test were plotted on top of each other as well as the theoretical results obtained from MATLAB for a 16-tooth idler. These results can be seen in Figure 5.10.

The mean results from each test appear to be quite similar, with both the shape and change in gear ratio being close. The max change in gear ratio for the aluminum idler, represented by the blue line in Figure 5.10, from 0.6916 to 0.7247 (0.0331), while slightly offset, is close in magnitude for the max gear ratio change of the plastic idler, represented by the green line in Figure 5.10, which ranged from 0.6909 to 0.7202 (0.0293). The difference of 0.0038 is only roughly 12.2% of the total change in gear ratio.

Based on these results, it can be concluded that the material of the sprocket makes only a minimal difference in the gear ratio change of the drivetrain. It would be expected that the material would be chosen for its durability rather than its effect on the drivetrain. It should be of note, however, that these results are limited; only two materials were tested. It is possible that other materials could have a greater effect on the drivetrain. It should also be noted that the two sprockets used had different tooth profiles.

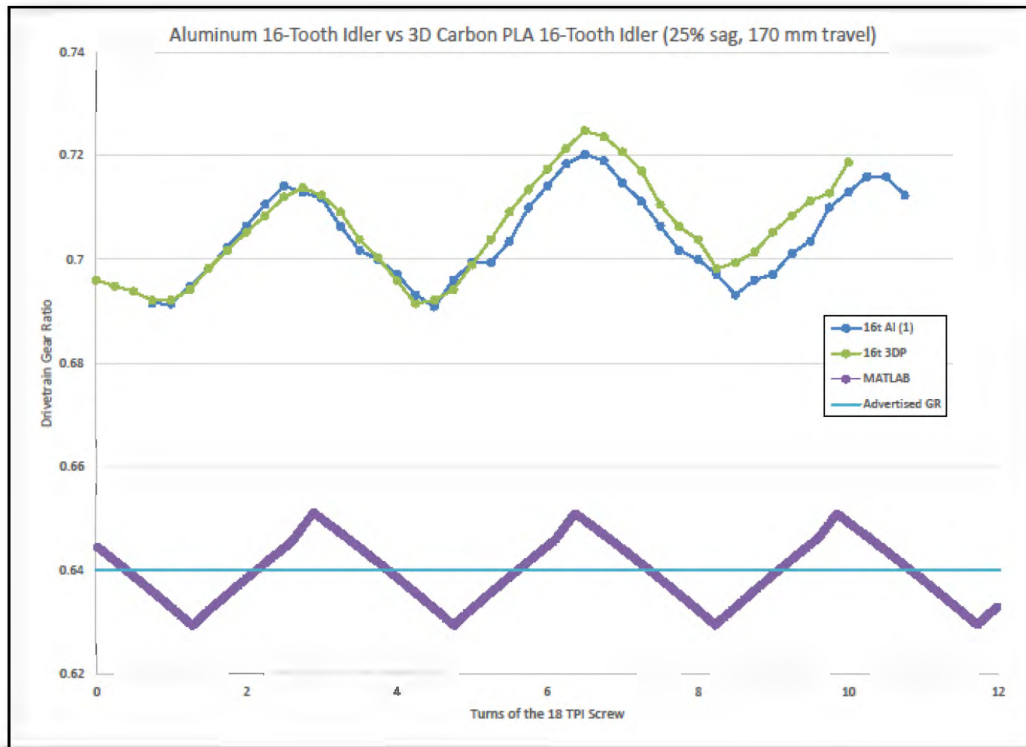


Figure 5.10 Idler Material Analysis Plot

5.1.5 – Chain-line Effects

To test the affect the chain line has on the change in gear ratio, the idler was offset by roughly 1” perpendicular to the chain-line and parallel to the floor to replicate a bad chain-line. Tests were performed with this set up and were then compare to the results from the normal chain-line tests. A 16-tooth 3D Carbon PLA sprocket, with a narrow-wide tooth profile was used, and both tests were performed at 25% sag.

The mean of the normal chain line results, represented by the green line in Figure 5.11, at the maximum change in gear ratio varies from 0.7137 to 0.6915 (0.0222), which is quite close to the max change in the theoretical data, obtained from MATLAB, at 0.0215. The mean results from the bad chain-line tests, represented by the blue line in Figure 5.11, shows a variation from 0.7254 to 0.6962 (0.0292).

It is apparent that the worse chain-line introduces a higher change in gear ratio than a normal one. Though the difference in the max variation, at 0.007, might not be considered significant, the test was done with only a 1” idler offset. Due to the geometry of the test

bench, this offset was the largest that could be used; it is quite likely that as the chain-line gets worse, the larger the max variation in gear ratio would also increase.

As well as producing worse rumbling in the chain, a bad chain-line would also result in worse frictional losses as well as increase the stress in the chain, reducing its life.

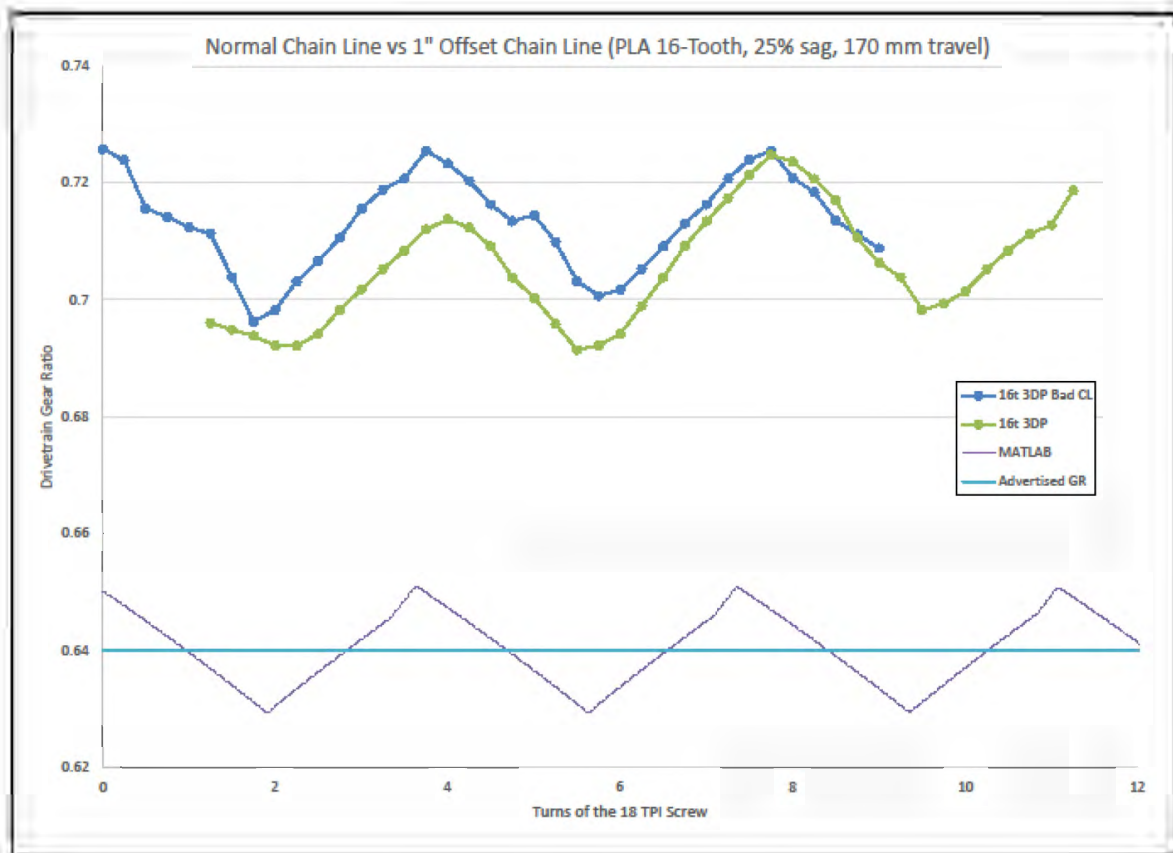


Figure 5.11 Chain-line Analysis Plot

5.1.6 – No Idler Test

As a final double check to the testing procedure of the test bench, and its validity, the design team ran a test with a shortened chain, and no idler in the drivetrain, essentially equivalent to a traditional bicycle drivetrain. This test was completed two times for minimal statistical verification, and the mean results between these two tests is shown below in Figure 5.12.

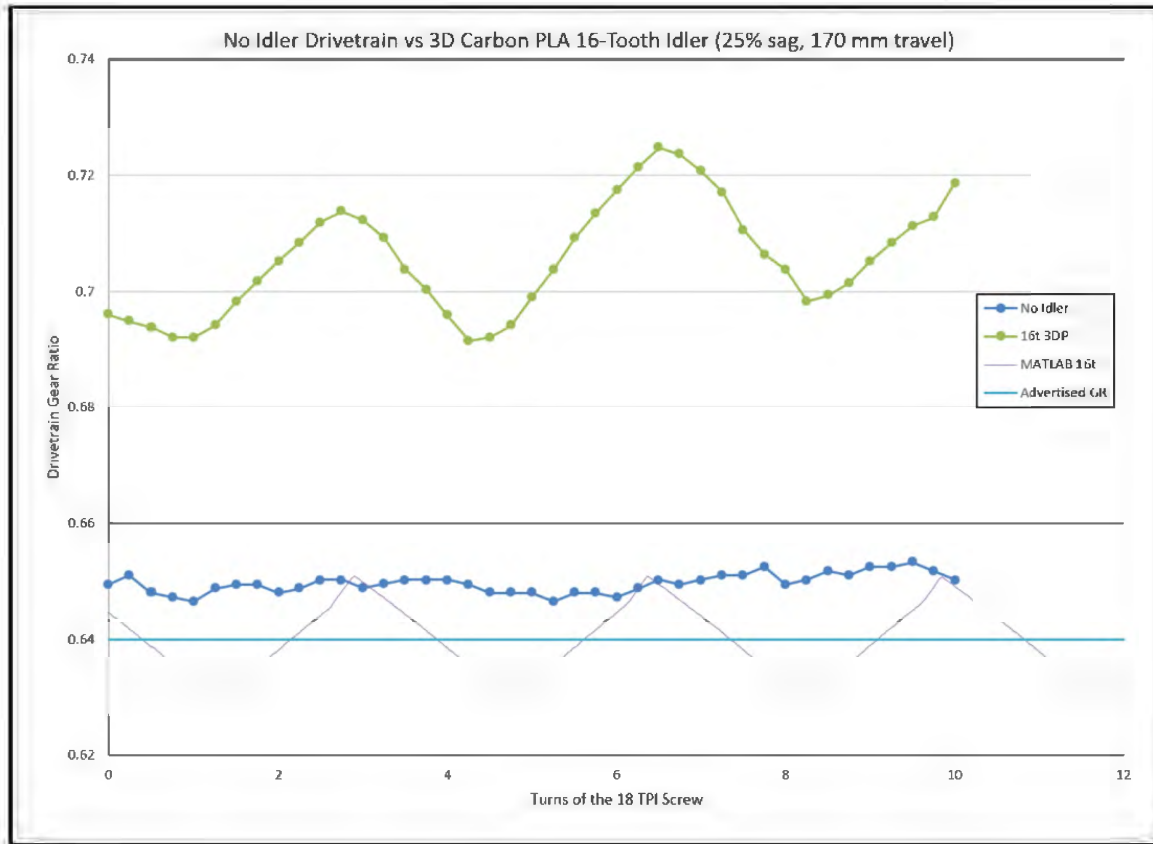


Figure 5.12 No Idler Analysis Plot

As can be seen in Figure 5.12, the no idler test, shown in dark blue, only varies a very slight amount. This variation is nearly impossible to view any profile as it is outside the usable resolution of the scale utilized. Regardless, the vibrations in this test as compared to the middle ground 16-tooth test, are insignificant, and are not comparable for this analysis.

It can also be noted, that with the removal of the idler, the offset due to system frictions is significantly reduced from the 16-tooth example. This is expected as the system is simplified and requires significantly less chain bending throughout the system resulting in an expected friction reduction.

5.2 – Project Difficulties

One difficulty with the project was the stiffness of the aluminum extrusion used in the test bench. The aluminum was used because it appeared to be an easy and available base material. However, after the first test at the initial weight of 50 lb., there was found to be significant deflection in some members of the bench. Specifically, the horizontal member which held the rear hub mounts as well as the vertical member which held the idler sprocket. Even with multiple braces these two members deflected significantly. The vertical member deflected 3/16" horizontally and the horizontal beam twisted allowing the hub to drop 3/16". This led to drift in measurements and inconsistent data. In the following tests the team elected to limit the maximum weight to 25lbs, eliminating any drift. When the weight was increased, this deflection was compensated for by moving the system the measured deflected amounts (although few tests were performed with this method).

Another difficulty was the friction inherent in the test bench. In the analytical model, there was no way to accurately estimate the amount of friction present in a real-world situation. Therefore, when the results from the analytical model and the physical test bench are compared there is an apparent offset between the two curves in all tests.

5.3 – Applicable Findings for Norco Bicycles

Through analyzing both the analytical and experimental data collected throughout the course of the project, the project team has found some results that would be applicable to Norco Bicycles.

First, it was determined that the rumbling phenomenon does indeed exist and should be considered when designing a high-pivot suspension bike. It was also concluded that the idler sprocket is mostly responsible for this phenomenon and that a designer should focus on it if they intend to minimize this effect.

Second, it was found that the most significant variable when it comes to the drivetrain rumbling is the tooth count of the idler. By selecting the proper number of teeth, significant

reductions in the rumbling can be achieved. The MATLAB analytical model and the test bench can be used to find the ideal tooth count for any bike frame layout.

If any other variables need to be analyzed, such as tooth profile or sprocket material, the test bench can be adapted to testing these variables. Note that the experiments performed over the course of the project were limited in scope, far more insight into the rumbling phenomenon could be gained by testing materials and tooth profiles not covered in this project.

6 – Conclusion

The hypothesis for this project was that either the chain-line, tooth profile, polygonal effect, or a combination of the three was the cause of the “rumbling” and inefficiencies felt when an idler was added to a generic bicycle drivetrain. The team also hypothesized that these problems could possibly be negated by optimizing idler size and position relative to other components.

The chain-line was found to increase frictional losses but did not contribute to any major gear ratio change. The material of the idler was also tested, and it was found to have no significant effect. The tooth profile, as well, did not have any appreciable effect on the gear ratio change.

The main contributor to the losses and the “rumbling” sensation was the polygonal effect of the idler. In the physical test with no idler installed, there was essentially no noticeable gear change as the system was rotated; however, in all other physical tests there was significant saw-tooth pattern to the changing gear ratios. The pattern produced in the physical tests can be directly compared to the same patterns found in the analytical model, which used the polygonal effect for its calculations. For this reason, it is safe to conclude that the polygonal effect of the idler is the cause of the “rumbling” sensation and inefficiencies.

It was also proven that the effects created by the idler can also be negated by changing the tooth count or repositioning it to optimize it for a given drivetrain. This is shown when comparing the 11-tooth physical tests to the 14-tooth physical tests. The 14-tooth idler offers a significant reduction in gear change as the system rotates versus the 11-tooth idler. The theory of repositioning is also proven in the physical tests and offered a slight reduction in gear change when optimized.

6.1 – Summary of Activity

The team began by conducting a literature survey to better understand the topics related to this project. The team researched the history of high-pivot mountain bikes, the reason for the addition of the idler to the drivetrain, and opinions on the “rumbling” and inefficiencies

associated with it. Next, the team looked at the more technical side of things including the polygonal nature of the chainrings, their changing gear ratios, and ways to reduce the effects.

Next, the analytical model was developed in MATLAB. The base code was written, and once a running model was completed, its outputs were analyzed to ensure they were reasonable. Optimization of the analytical model then began, and tests were run for various positions, idler sizes, and cassette sizes. Excel documents were then created to better store and visualize the data.

Next the physical test bench was developed to verify the MATLAB results and test other theories not possible in the analytical model. Multiple design sessions were conducted to better define what the test bench needed to accomplish and how it would be manufactured. Eventually, a design was finalized, and manufacturing could begin. Parts were ordered from multiple vendors and the test bench was quickly assembled.

Finally, the physical test bench was used to perform multiple tests with different idler sizes and different positions. The data was collected and compared to the analytical results to confirm their validity.

6.2 – Discrepancies in Project Outputs

The analytical results showed a lower gear ratio than actually produced in the physical test bench. This is reflected in Figure 5.2. Two possible explanations for this are that, one, the scale is not accurate and the more likely reason being friction generated in the system. The analytical model does not take into account any friction generated by the bicycle drivetrain, bearings, steel cables, chain, and other sources found in the system. This results in less weight being shown on the scale than predicted and the gear ratio being calculated with augmented values. This friction is believed to be the most probable cause for both the offset present on plots comparing simulation to real world gear ratio tests (Such as Figure 5.2) in addition to possible hysteresis appearing on plots.

6.3 – Future Work Recommendations

In the future it would be recommended to create a stiffer frame for the test bed. When the test bed was loaded with the weight that the team intended on using there was significant deflection resulting in drifting data. Therefore, the weight was reduced to decrease deflection and eliminate any drift in measurements. The reduction in weight, however, reduced the accuracy of the scale slightly, fortunately, the results were still acceptable for most tests.

Another future consideration would be to use a strain gauge or other force sensor to automate the data input with programming. This would significantly reduce the time to run a test and would result in more accurate data as there would not be such large movement gaps between data points. The strain gauge or force sensor could also be more accurate than the scale used for these tests because the quality of the scale's internal workings is unclear and could be improved with more high-end components.

Another future consideration could be focusing on eliminating as much friction as possible in the system to increase the accuracy of the test bed in relation to the analytical model. Low friction bearings and oiling cable guides for the steel cables would both be possibilities for reducing friction.

One last future consideration would be to somehow motorize the movement of the aluminum extrusions and other parts. If a system similar to a 3D printer was utilized the coordinates of each part could be inputted into a program and the test bed would move to its new position while displaying them on digital read outs. This would significantly increase the accuracy of the test bed's mounting locations as well as reduce setup times significantly.

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-

Appendix A – MATLAB Scripts

A.1 – List of MATLAB Scripts with Brief Descriptions

- **CreateCircle:** Creates a list of XY coordinates to represent a given tooth number bicycle sprocket on a 2D plane.
 - **FindTangent_IC:** Determines the active tangent teeth between the idler and cassette sprockets given an array of sprocket points on a 2D plane.
 - **FindTangent_KI:** Determines the active tangent teeth between the crank and cassette sprockets given an array of sprocket points on a 2D plane.
 - **Init_Position_IC:** Rotates the current cassette points about the cassette center until a position is achieved which allows a one-half inch pitch chain to fit between the tangent points of the cassette and idler.
 - **Init_Position_KI:** Rotates the current crank points about the crank center until a position is achieved which allows a one-half inch pitch chain to fit between the tangent points of the crank and idler.
 - **SpinRatio:** Determines the series of gear ratios between each sprocket given a rotational position by determining the active apparent radius of each tangent tooth between the crank and idler sprockets, and the idler and cassette sprockets.
-

- **rotate:** Rotates the entire chain-line given an array of points for all 3 sprockets, current gear ratios, and an increment of crank sprocket rotation in degrees.
 - **MoveIdler:** Moves the position of the idler sprocket tooth array to ensure chain-line passes through same bicycle position as the currently specified 16 tooth idler sprocket given a new idler tooth value and an approximately 45-degree pivot location.
 - **Main_Static:** Main script, executes a single simulation with given locations and tooth counts for the crank, idler, and cassette sprockets (at bicycle static (no rider) position), along with rotational amount for crank. Outputs overall gear ratio values over specified rotation.
 - **Main_Static_XYMove:** Main script, executes a series of simulations with given locations and tooth counts for the crank, idler, and cassette sprockets (at bicycle static (no rider) position), along with rotational amount for crank, and a specified range of x (horizontal) position and y (vertical) variation. Outputs a 2D array of maximum gear ratio variation values over the field.
 - **Main_Static_tSweep:** Main script, executes a series of simulations with given locations and tooth counts for the crank, idler, and cassette sprockets (at bicycle static (no rider) position), along with rotational amount for crank, and a specified range of idler tooth values. Outputs a list of maximum gear ratio variation values for each idler.
 - **Main_Sag:** Main script, executes a single simulation with given locations and tooth counts for the crank, idler, and cassette sprockets (at bicycle sag (25% of 170mm travel) position), along with rotational amount for crank. Outputs overall gear ratio values over specified rotation.
-

- **Main_Sag_XYMove:** Main script, executes a series of simulations with given locations and tooth counts for the crank, idler, and cassette sprockets (at bicycle sag (25% of 170mm travel) position), along with rotational amount for crank, and a specified range of x (horizontal) position and y (vertical) variation. Outputs a 2D array of maximum gear ratio variation values over the field.
- **Main_Sag_tSweep:** Main script, executes a series of simulations with given locations and tooth counts for the crank, idler, and cassette sprockets (at bicycle sag (25% of 170mm travel) position), along with rotational amount for crank, and a specified range of idler tooth values. Outputs a list of maximum gear ratio variation values for each idler.
- **Load_Cell_Calc:** Determines the minimum allowable mass for the weight being used on the test bench. Calculates the change in weight from the maximum overall gear ratio to the minimum overall gear ratio, at both the worst- and best- case scenarios.

A.2 – Copy of MATLAB Scripts

CreateCircle:

```
function [circle,r] = Create_Circle(P,N)
%Creates an array of points forming a circle with a radius r

theta = pi/2; %Initial theta

r = P*(sin((pi - (2*pi/N))/2))/sin(2*pi/N); %Calculate the pitch radius

circle = ones(N,3); %Create array of points (x,y,theta)

circle(1,1) = r*cos(theta); %Calculate x of first tooth
circle(1,2) = r*sin(theta); %Calculate y of the first tooth
circle(1,3) = theta*(180/pi); %Calculate theta of the first tooth

%Calculate x, y, and theta for each tooth on the ring
for i = 2:N
    theta = (2*pi/N) + theta; %Increment theta
```

```
%Set theta back to zero after 2*Pi reached
if theta >= 2*pi
    theta = theta - 2*pi;
end

circle(i,1) = r*cos(theta); %Calculate x of tooth i
circle(i,2) = r*sin(theta); %Calculate y of tooth i
circle(i,3) = theta*(180/pi); %Calculate theta of tooth i

end
end
```

FindTangent_IC:

```
function [Tangent] =
FindTangent_IC(Ring1, Ring2, N1, N2, Idler_OS, Cassette_OS)
%Calculates the tangent line between the idler and the cassette

Y = 0; %Stores maximum x at y = 8
Tangent = ones(2,2); %Tangent line array

for i = 1:N1
    for j = 1:N2
        m = (Ring2(j,2)-Ring1(i,2))/(Ring2(j,1)-Ring1(i,1)); %Calculate
slope of connecting line
        b = (Ring1(i,2))-(m*Ring1(i,1)); %Calculate y intercept

        MP = Idler_OS(1)+Cassette_OS(1)/2; %Midpoint between the two rings

        y = m*MP + b; %Calculate the x position at y = MP

        %Find which x at y = MP is farthest to the left, this is the
tangent line
        if y > Y
            Y = y;
            m_t = m;
            x1 = Ring1(i,1);
            y1 = Ring1(i,2);
            x2 = Ring2(j,1);
            y2 = Ring2(j,2);
            b_t = b;
        end
    end
end

%Assign the coordinates of the endpoints to the array
Tangent(1,1) = x1;
Tangent(1,2) = y1;
Tangent(2,1) = x2;
Tangent(2,2) = y2;

% %Find step size (1/100 of total length)
% step = (x2 - x1)/100;
```

```

%
% %Calculate each point on the line
% for i = 0:100
%     x = (step*i)+x1;
%     y = x*m_t+b_t;
%
%     Tangent(i+1,1) = x;
%     Tangent(i+1,2) = y;
% end
end

```

FindTangent_KI:

```

function [Tangent] = FindTangent_KI(Ring1, Ring2, N1, N2, Idler_OS)
%Calculates the tangent line between the crank and the idler

X = 0; %Stores the maximum x at y = 4
Tangent = ones(2,2); %Tangent line array

for i = 1:N1
    for j = 1:N2
        m = (Ring2(j,2)-Ring1(i,2))/(Ring2(j,1)-Ring1(i,1)); %Calculate
slope of connecting line
        b = (Ring1(i,2))-(m*Ring1(i,1)); %Calculate y intercept

        MP = Idler_OS(2)/2; %Midpoint bewtween the two rings

        x = (MP - b)/m; %Calculate the x position at y = 4

        %Find which x @ y = 4 is farthest to the right, this is the tangent
line
        if x > X
            X = x;
            m_t = m;
            x1 = Ring1(i,1);
            y1 = Ring1(i,2);
            x2 = Ring2(j,1);
            y2 = Ring2(j,2);
            b_t = b;
        end
    end
end

%Assign the coordinates of the endpoints to the array
Tangent(1,1) = x1;
Tangent(1,2) = y1;
Tangent(2,1) = x2;
Tangent(2,2) = y2;

% %Calculate the step size (1/100 of total length)
% step = (x2 - x1)/100;
%
% %Calculate each point on the line
% for i = 0:100
%     x = (step*i)+x1;

```

```
%    y = x*m_t+b_t;
%
%    Tangent(i+1,1) = x;
%    Tangent(i+1,2) = y;
% end
end
```

Init_Position_IC:

```
function [Cassette, theta_cassette] = Init_Position_IC(Idler,Cassette,
dtheta_init,N_i,N_c,TOL,r_c,GR_kc,Idler_OS,Cassette_OS)
%UNTITLED Summary of this function goes here
%    Detailed explanation goes here

k = 0;

T = FindTangent_IC(Idler,Cassette,N_i,N_c,Idler_OS,Cassette_OS); %Find
initial tangent line

[n(1),n(2)] = find(Cassette == Cassette(2,3)); %Find coordinates of tooth
at max theta

while k == 0

T(2,1) = Cassette(n(1),1); %Assign x coordinate of tooth
T(2,2) = Cassette(n(1),2); %Assign y coordinate of tooth
theta_cassette = Cassette(n(1),3);%Assign theta of tooth

if theta_cassette > 360
    theta_cassette = theta_cassette - 360;
elseif theta_cassette < 0
    theta_cassette = theta_cassette + 360;
end

L = sqrt((T(2,2) - T(1,2))^2 + (T(2,1) - T(1,1))^2); %Calculate length of
line

%Check to see if the length of the chain is divisible by 1/2"
if abs(rem(L,0.5)) > TOL
    Cassette = rotate(Cassette,-dtheta_init,N_c,r_c,GR_kc);
else
    k = 1;
end

end

%Tan = FindTangent_IC(Idler,Cassette,N_i,N_c);

% if Tan(2,2) - T(2,2) < 0.001
%     'Line is tangent'
% end

end
```

Init_Position_KI:

```

function [Idler, theta_idler] = Init_Position_KI(Crank,Idler,
dtheta_init,N_k,N_i,TOL,r_i,GR_ki,Idler_OS)
%UNTITLED Summary of this function goes here
% Detailed explanation goes here

k = 0;

T = FindTangent_KI(Crank,Idler,N_k,N_i,Idler_OS); %Find initial tangent
line

[n(1),n(2)] = find(Idler == min(Idler(1:N_i,3))); %Find coordinates of
tooth at max theta

while k == 0

T(2,1) = Idler(n(1),1); %Assign x coordinate of tooth
T(2,2) = Idler(n(1),2); %Assign y coordinate of tooth
theta_idler = Idler(n(1),3);%Assign theta of tooth

%Set theta back to 0 if it becomes greater than 360
if theta_idler > 360
    theta_idler = theta_idler - 360;
%Set theta to 360 if it becomes less than 0
elseif theta_idler < 0
    theta_idler = theta_idler + 360;
end

L = sqrt((T(2,2) - T(1,2))^2 + (T(2,1) - T(1,1))^2); %Calculate length of
line

%Check to see if the length of the chain is divisible by 1/2"
if abs(rem(L,0.5)) > TOL
    Idler = rotate(Idler,-dtheta_init,N_i,r_i,GR_ki);
else
    k = 1;
end

end

%Calculate coordinates of tangent line
%Tan = FindTangent_KI(Crank,Idler,N_k,N_i);

%Display message if tangent line has same coordinates as point calculated
above
% if Tan(2,2) - T(2,2) < 0.001
%     'Line is tangent'
% end

end

```

SpinRatio:

```

function [SR, ra_I, ra_O] = SpinRatio(T,Idler_OS, Other_OS)

```

```
%This function will calculate the gear ratio between the idler sprocket and
%one other sprocket(chairing or cassette)
%Inputs:  T          - Location of teeth in use
%          nI         - number of teeth on the Idler
%          n          - Number of teeth on *other sprocket
%          Idler_OS   - Center coordinates of idler
%          Other_OS   - Center coordinates of *other sprocket

%Outputs: R          - the 'gear' ratio of idler/*other turns
%          ra_I       - the apparent radius of the idler for this sprocket
%          ra_O       - the apparent radius of the other sprocket
%Assumptions: Idler is always higher than top of cassette
% *****

clc
%temp values 4 testing*****
% %T= [-1.5979 8.2452;-17.0096 3.0430];%cassette
% T =
[2.35642340055245,0.976062531202507;0.0200642868907273,7.56088349729150];%
crank
% nI = 14;
% %n = 27;%testing with cassette
% n = 32; %testing with crank
% Idler_OS = [-1.0618,7.2579]; %Coordinates of idler axle (offset)
% %Other_OS = [-16.5079,0.9488];%cassette
% Other_OS = [0,0]; %crank
%*****
step = .1;
%determine which point is on idler
if T(1,2)>T(2,2)
    %1st coordinates higher y therefore is idler
    pI = T(1,:);%idler position
    pO = T(2,:);%other position
else
    %2nd coordinates higher y therefore is idler
    pI = T(2,:);%idler tooth position
    pO = T(1,:); %other tooth position
end
%find chain line m and b (y=mx+b)
mChain = (pI(2)-pO(2))/(pI(1)-pO(1));
bChain = pI(2)-mChain*pI(1);

%convert to angle
thetaChain = atan(mChain);

%change negative slope to equivalent positive slope
if thetaChain<0
    thetaChain = thetaChain+pi;
end

%find slopes of radial lines
mI = (pI(2)- Idler_OS(2))/(pI(1)- Idler_OS(1)) ;%tooth radial line idler
mO = (pO(2)- Other_OS(2))/(pO(1)- Other_OS(1)) ;%tooth radial line other

%convert to angle
thetaI = atan(mI);
thetaO = atan(mO);
```

```

%change negative slopes to equivalent positive slope
if thetaI<0
    thetaI = thetaI+pi;
end
if thetaO<0
    thetaO = thetaO+pi;
end

%initialize apparent radius variables
ra_I = 1000; ra_O = 10000;

%sweep through idler angle range***
startAngle = 180/pi*(thetaI-pi/4);
stopAngle = 180/pi*(thetaI+pi/8);
j=0;
R=0;
for i = startAngle:step: stopAngle
    %convert to radians (m = dy/dx => theta = tan^-1(m))
    i;
    j=j+1;
    theta = i*pi/180;
    %find slope
    m = tan(theta);
    %find b intercepts for the idler radial lines
    bI = Idler_OS(2) - m*Idler_OS(1);

    %find interception points of chain and idler radial line
    x = (bChain-bI)/(m-mChain);
    y = m*x+bI;

    R(j) = sqrt((x-Idler_OS(1))^2+(y-Idler_OS(2))^2);
    if R(j)<ra_I %find minimum radius along angle sweep
        ra_I = R(j);
    end
end
%plot(R)

%repeat for *other sprocket apparent radius
%sweep through other sprocket angle range***
startAngle = 180/pi*(thetaO-pi/4);
stopAngle = 180/pi*(thetaO+pi/4);
j=0;

R=0;
for i = startAngle:step: stopAngle
    %convert to radians (m = dy/dx => theta = tan^-1(m))
    i;
    j=j+1;
    theta = i*pi/180;
    %find slope
    m = tan(theta);
    %find b intercepts for the idler radial lines
    bO = Other_OS(2) - m*Other_OS(1);

    %find interception points of chain and other radial line
    x = (bChain-bO)/(m-mChain);

```

```
y = m*x+b0;

R(j) = sqrt((x-Other_OS(1))^2+(y-Other_OS(2))^2);
if R(j)<ra_O %find minumum radius along angle sweep
    ra_O = R(j);
end
end
%plot(R)
SR = (ra_O/ra_I);
end
```

Rotate:

```
function [Circle] = rotate(Circle,deg,N,r,GR)
%UNTITLED2 Summary of this function goes here
% Detailed explanation goes here

theta2 = Circle(1:N,3)-deg*GR; %Create array of new thetas

%Reset theta to 360 deg when it passes 0 deg
for i = 1:N
    if theta2(i) < 0
        theta2(i) = 360 + theta2(i);
    end
end

%Calculate new x and y for each point
for i=1:N
    %Calculate change in x
    dx = r*cos(theta2(i)*(pi/180)) - r*cos(Circle(i,3)*(pi/180));
    %Calculate change in y
    dy = r*sin(theta2(i)*(pi/180)) - r*sin(Circle(i,3)*(pi/180));

    %Calculate new x
    Circle(i,1) = Circle(i,1) + dx;
    %Calculate new y
    Circle(i,2) = Circle(i,2) + dy;
    %Assign new theta
    Circle(i,3) = theta2(i);
end

end
```

Moveldler:

```
function [Idler_OS] = MoveIdler(Idler_OS,N_i)
%MOVEILDER - A function that move the position of the idler to ensure the
%chainline passes through the same position as it would with the
originally
%spec'd 16t idler (Assumes standard 0.5" pitch, therefore units in inches)
% x16 = x position of 16t passthrough
% y16 = y position of 16t passthrough
% Rp16 = 16t pitch radius
% xN = x position of current N idler passthrough before move
```

```
%   yN  = y position of current N idler passthrough before move
%   Rp  = N tooth pitch radius
%   dx   = Amount to move center x direction
%   dy   = Amount to move center y direction
```

```
%Calc. 16t position passthrough (assume ~45 degrees on sprocket)
Rp16 = 0.5/(sin(pi/16))*0.5;
x16 = Rp16*cos(45*pi/180);
y16 = x16;
```

```
%Calc. N teeth position passthrough
Rp = 0.5/(sin(pi/N_i))*0.5;
xN = Rp*cos(45*pi/180);
yN = xN;
```

```
dx = xN-x16;
dy = yN-y16;
```

```
Idler_OS = Idler_OS - [dx, dy];
end
```

Main_Static:

```
%//////////////////////////////////////////
%// K = CRANK   I = IDLER   C = CASSETTE //
%//////////////////////////////////////////
```

```
N_k = 32; %Number of teeth on crank
N_i = 12; %Number of teeth on idler
N_c = 50; %Number of teeth on cassette
P = 0.5; %Pitch
dtheta_init = 0.001; %Change in angle to find initial position of
sprockets
TOL = 0.001; %Tolerance for finding initial sprocket positions
dtheta = 0.1; %Change in angle
Crank_OS = [0,0];
Idler_OS = [-1.0618,7.2579]; %Coordinates of idler axle (offset)
Cassette_OS = [-16.5079,0.9488]; %Coordinates of cassette axle (offset)
steps = 45/dtheta; %Number of steps
```

```
%Move idler so chainline passes through same position
Idler_OS = MoveIdler(Idler_OS,N_i);
```

```
%initialize arrays
KI_Length = ones(steps,3);
IC_Length = ones(steps,2);
ra_IK = zeros(steps,1);
ra_K = zeros(steps,1);
ra_IC = zeros(steps,1);
ra_C = zeros(steps,1);
```

```
[Crank,r_k] = Create_Circle(P,N_k); %Create initial array of points for
crank
```

```
[Idler,r_i] = Create_Circle(P,N_i); %Create initial array of points for
idler

[Cassette,r_c] = Create_Circle(P,N_c); %Create initial array of points for
cassette

GR_ki = r_k/r_i; %crank-idler gear ratio
GR_kc = r_k/r_c; %crank-cassette gear ratio

%Adjust idler points with offset
for i = 1:N_i
    Idler(i,1) = Idler(i,1) + Idler_OS(1);
    Idler(i,2) = Idler(i,2) + Idler_OS(2);
end

%Adjust cassette points with offset
for i = 1:N_c
    Cassette(i,1) = Cassette(i,1) + Cassette_OS(1);
    Cassette(i,2) = Cassette(i,2) + Cassette_OS(2);
end

[Idler,idler_theta] =
Init_Position_KI(Crank,Idler,dtheta_init,N_k,N_i,TOL,r_i,GR_ki,Idler_OS);
[Cassette, theta_cassette] =
Init_Position_IC(Idler,Cassette,dtheta_init,N_i,N_c,TOL,r_c,GR_kc,Idler_OS
,Cassette_OS);

%Rotate crank full rotation in steps of dtheta

for i = 1:steps

T1 = FindTangent_KI(Crank,Idler,N_k,N_i,Idler_OS); %Calculate the tangent
line between the crank and the idler

T2 = FindTangent_IC(Idler,Cassette,N_i,N_c,Idler_OS,Cassette_OS);
%Calculate the tangent line between the idler and the cassette

%Plot points
figure(1)
drawnow
plot(Crank(1:N_k,1),Crank(1:N_k,2), 'r.', Idler(1:N_i,1), Idler(1:N_i,2), 'b.'
,Cassette(1:N_c,1),Cassette(1:N_c,2), 'm.', T1(1:2,1), T1(1:2,2), T2(1:2,1), T2
(1:2,2));
axis equal
grid on

KI_Length(i,1) = dtheta*i; %Turn angle of crank
KI_Length(i,2) = sqrt((T1(2,1)-T1(1,1))^2 + (T1(2,2)-T1(1,2))^2);
%Calculate length of KI tangent line
KI_Length(i,3) = atan((T1(2,2)-T1(1,2))/(T1(2,1)-T1(1,1)));

IC_Length(i,1) = dtheta*i; %Turn angle of crank
IC_Length(i,2) = sqrt((T2(2,1)-T2(1,1))^2 + (T2(2,2)-T2(1,2))^2);
%Calculate length of IC tangent line
```

```

%calculate live gear ratios
[GR_ki(i), ra_IK(i), ra_K(i)] = SpinRatio(T1,Idler_OS, Crank_OS); %crank-
idler gear ratio
[GR_ic(i), ra_IC(i), ra_C(i)] = SpinRatio(T2,Idler_OS, Cassette_OS);
%crank-cassette gear ratio
GR_kc(i) = GR_ki(i)/GR_ic(i);

if i~=1
Crank = rotate(Crank,dtheta,N_k,r_k,1); %Rotate crank

Idler = rotate(Idler,dtheta,N_i,r_i,GR_ki); %Rotate idler relative to
crank

Cassette = rotate(Cassette,dtheta,N_c,r_c,GR_kc); %Rotate cassette
relative to crank
end
end

% %Plot change in length of tangent lines
% figure(2)
%
plot(KI_Length(1:steps,1),KI_Length(1:steps,2),IC_Length(1:steps,1),IC_Len
gth(1:steps,2));
% legend('Crank/Idler','Idler/Cassette','Location','east');
%
% figure(3)
% plot(KI_Length(1:steps,1),(180/pi)*KI_Length(1:steps,3));

%plot apparent radai of the three sprockets
% figure(2)
% hold on
% plot(ra_IK,'c')
% plot(ra_IC,'m')
% plot(ra_K,'b')
% plot(ra_C,'r')
% legend('Idler, Crank Side','Idler, Cassette Side','Crank','Cassette');
% title("Apparent Radius' of Sprockets");
% ylabel("Apparent Radius, [in]")
% hold off
figure(2)
plot(GR_kc)
title('Gear Ratio Variation');
ylabel('Overall Gear Ratio');

```

Main_Static_XYMove:

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%// K = CRANK   I = IDLER   C = CASSETTE //
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

N_k = 32; %Number of teeth on crank
N_i = 16; %Number of teeth on idler
N_c = 50; %Number of teeth on cassette

```

```
P = 0.5; %Pitch
dtheta_init = 0.001; %Change in angle to find initial position of
sprockets
TOL = 0.001; %Tolerance for finding initial sprocket positions
dtheta = 0.1; %Change in angle
Crank_OS = [0,0];
Idler_OS = [-1.0618,7.2579]; %Coordinates of idler axle (offset)
Cassette_OS = [-16.5079,0.9488]; %Coordinates of cassette axle (offset)
steps = 20/dtheta; %Number of steps
xy_Step = 0.05; %step size for moving in xy [inches]
distXY = .5; %distance to travel from current position in +x,-x,+y,-y
directions
xy_Steps = 2*distXY/xy_Step;

%Move idler so chainline passes through same position
Idler_OS = MoveIdler(Idler_OS,N_i);

for n=0:xy_Steps%define position range array
    xy(n+1,[1 2]) = Idler_OS -[distXY distXY] + n*[xy_Step xy_Step];
end

for n=0:xy_Steps %loop through y
    Idler_OS(1,2) = xy(n+1,2);

    for m=0:xy_Steps %loop through x
        Idler_OS(1,1) = xy(m+1,1);

%initialize arrays
KI_Length = ones(steps,3);
IC_Length = ones(steps,2);
ra_IK = zeros(steps,1);
ra_K = zeros(steps,1);
ra_IC = zeros(steps,1);
ra_C = zeros(steps,1);

[Crank,r_k] = Create_Circle(P,N_k); %Create initial array of points for
crank

[Idler,r_i] = Create_Circle(P,N_i); %Create initial array of points for
idler

[Cassette,r_c] = Create_Circle(P,N_c); %Create initial array of points for
cassette

GR_ki = r_k/r_i; %crank-idler gear ratio
GR_kc = r_k/r_c; %crank-cassette gear ratio

%Adjust idler points with offset
for i = 1:N_i
    Idler(i,1) = Idler(i,1) + Idler_OS(1);
    Idler(i,2) = Idler(i,2) + Idler_OS(2);
end

%Adjust cassette points with offset
for i = 1:N_c
    Cassette(i,1) = Cassette(i,1) + Cassette_OS(1);
```

```

    Cassette(i,2) = Cassette(i,2) + Cassette_OS(2);
end

[Idler,idler_theta] =
Init_Position_KI(Crank,Idler,dtheta_init,N_k,N_i,TOL,r_i,GR_ki,Idler_OS);
[Cassette, theta_cassette] =
Init_Position_IC(Idler,Cassette,dtheta_init,N_i,N_c,TOL,r_c,GR_kc,Idler_OS
,Cassette_OS);

%Rotate crank full rotation in steps of dtheta

for i = 1:steps

T1 = FindTangent_KI(Crank,Idler,N_k,N_i,Idler_OS); %Calculate the tangent
line between the crank and the idler

T2 = FindTangent_IC(Idler,Cassette,N_i,N_c,Idler_OS,Cassette_OS);
%Calculate the tangent line between the idler and the cassette

%Plot points
figure(1)
%drawnow
plot(Crank(1:N_k,1),Crank(1:N_k,2),'r.',Idler(1:N_i,1),Idler(1:N_i,2),'b.'
,Cassette(1:N_c,1),Cassette(1:N_c,2),'m.',T1(1:2,1),T1(1:2,2),T2(1:2,1),T2
(1:2,2));
axis equal
grid on

KI_Length(i,1) = dtheta*i; %Turn angle of crank
KI_Length(i,2) = sqrt((T1(2,1)-T1(1,1))^2 + (T1(2,2)-T1(1,2))^2);
%Calculate length of KI tangent line
KI_Length(i,3) = atan((T1(2,2)-T1(1,2))/(T1(2,1)-T1(1,1)));

IC_Length(i,1) = dtheta*i; %Turn angle of crank
IC_Length(i,2) = sqrt((T2(2,1)-T2(1,1))^2 + (T2(2,2)-T2(1,2))^2);
%Calculate length of IC tangent line

%calculate live gear ratios
[GR_ki(i), ra_IK(i), ra_K(i)] = SpinRatio(T1,Idler_OS, Crank_OS); %crank-
idler gear ratio
[GR_ic(i), ra_IC(i), ra_C(i)] = SpinRatio(T2,Idler_OS, Cassette_OS);
%crank-cassette gear ratio
GR_kc(i) = GR_ki(i)/GR_ic(i);

if i~=1
Crank = rotate(Crank,dtheta,N_k,r_k,1); %Rotate crank

Idler = rotate(Idler,dtheta,N_i,r_i,GR_ki); %Rotate idler relative to
crank

Cassette = rotate(Cassette,dtheta,N_c,r_c,GR_kc); %Rotate cassette
relative to crank
end
end

```

```
% %Plot change in length of tangent lines
% figure(2)
%
plot(KI_Length(1:steps,1),KI_Length(1:steps,2),IC_Length(1:steps,1),IC_Length(1:steps,2));
% legend('Crank/Idler','Idler/Cassette','Location','east');
%
% figure(3)
% plot(KI_Length(1:steps,1),(180/pi)*KI_Length(1:steps,3));

%plot apparent radai of the three sprockets
% figure(2)
% hold on
% plot(ra_IK,'c')
% plot(ra_IC,'m')
% plot(ra_K,'b')
% plot(ra_C,'r')
% legend('Idler, Crank Side','Idler, Cassette Side','Crank','Cassette');
% title("Apparent Radius' of Sprockets");
% ylabel("Apparent Radius, [in]")
% hold off

GR(n+1,m+1) = max(GR_kc)- min(GR_kc);

    end
end
```

Main_Static_tSweep:

```
%////////////////////////////////////////
%// K = CRANK I = IDLER C = CASSETTE //
%////////////////////////////////////////

N_k = 32; %Number of teeth on crank
N_i = 16; %Number of teeth on idler
N_c = 50; %Number of teeth on cassette
P = 0.5; %Pitch
dtheta_init = 0.001; %Change in angle to fine initial position of
sprockets
TOL = 0.001; %Tolerance for finding initial sprocket positions
dtheta = 0.1; %Change in angle
Crank_OS = [0,0];
Idler_OS_old = [-1.0618,7.2579]; %Coordinates of idler axle (offset)
Cassette_OS = [-16.5079,0.9488]; %Coordinates of cassette axle (offset)
steps = 30/dtheta; %Number of steps

%Move idler so chainline passes through same position
Idler_OS = MoveIdler(Idler_OS_old,N_i);

w=0;
for N_i = 10:25
%Move idler so chainline passes through same position
```

```

Idler_OS = MoveIdler(Idler_OS_old,N_i);
    w = w+1;

%initialize arrays
KI_Length = ones(steps,3);
IC_Length = ones(steps,2);
ra_IK = zeros(steps,1);
ra_K = zeros(steps,1);
ra_IC = zeros(steps,1);
ra_C = zeros(steps,1);

[Crank,r_k] = Create_Circle(P,N_k); %Create initial array of points for
crank

[Idler,r_i] = Create_Circle(P,N_i); %Create initial array of points for
idler

[Cassette,r_c] = Create_Circle(P,N_c); %Create initial array of points for
cassette

GR_ki = r_k/r_i; %crank-idler gear ratio
GR_kc = r_k/r_c; %crank-cassette gear ratio

%Adjust idler points with offset
for i = 1:N_i
    Idler(i,1) = Idler(i,1) + Idler_OS(1);
    Idler(i,2) = Idler(i,2) + Idler_OS(2);
end

%Adjust cassette points with offset
for i = 1:N_c
    Cassette(i,1) = Cassette(i,1) + Cassette_OS(1);
    Cassette(i,2) = Cassette(i,2) + Cassette_OS(2);
end

[Idler,idler_theta] =
Init_Position_KI(Crank,Idler,dtheta_init,N_k,N_i,TOL,r_i,GR_ki,Idler_OS);
[Cassette, theta_cassette] =
Init_Position_IC(Idler,Cassette,dtheta_init,N_i,N_c,TOL,r_c,GR_kc,Idler_OS
,Cassette_OS);

%Rotate crank full rotation in steps of dtheta

for i = 1:steps

T1 = FindTangent_KI(Crank,Idler,N_k,N_i,Idler_OS); %Calculate the tangent
line between the crank and the idler

T2 = FindTangent_IC(Idler,Cassette,N_i,N_c,Idler_OS,Cassette_OS);
%Calculate the tangent line between the idler and the cassette

%Plot points
figure(1)
drawnow

```

```
plot(Crank(1:N_k,1),Crank(1:N_k,2),'r.',Idler(1:N_i,1),Idler(1:N_i,2),'b.'
,Cassette(1:N_c,1),Cassette(1:N_c,2),'m.',T1(1:2,1),T1(1:2,2),T2(1:2,1),T2
(1:2,2));
axis equal
grid on

KI_Length(i,1) = dtheta*i; %Turn angle of crank
KI_Length(i,2) = sqrt((T1(2,1)-T1(1,1))^2 + (T1(2,2)-T1(1,2))^2);
%Calculate length of KI tangent line
KI_Length(i,3) = atan((T1(2,2)-T1(1,2))/(T1(2,1)-T1(1,1)));

IC_Length(i,1) = dtheta*i; %Turn angle of crank
IC_Length(i,2) = sqrt((T2(2,1)-T2(1,1))^2 + (T2(2,2)-T2(1,2))^2);
%Calculate length of IC tangent line

%calculate live gear ratios
[GR_ki(i), ra_IK(i), ra_K(i)] = SpinRatio(T1,Idler_OS, Crank_OS); %crank-
idler gear ratio
[GR_ic(i), ra_IC(i), ra_C(i)] = SpinRatio(T2,Idler_OS, Cassette_OS);
%crank-cassette gear ratio
GR_kc(i) = GR_ki(i)/GR_ic(i);

if i~=1
Crank = rotate(Crank,dtheta,N_k,r_k,1); %Rotate crank

Idler = rotate(Idler,dtheta,N_i,r_i,GR_ki); %Rotate idler relative to
crank

Cassette = rotate(Cassette,dtheta,N_c,r_c,GR_kc); %Rotate cassette
ralative to crank
end
end

% %Plot change in length of tangent lines
% figure(2)
%
plot(KI_Length(1:steps,1),KI_Length(1:steps,2),IC_Length(1:steps,1),IC_Len
gth(1:steps,2));
% legend('Crank/Idler','Idler/Cassette','Location','east');
%
% figure(3)
% plot(KI_Length(1:steps,1),(180/pi)*KI_Length(1:steps,3));

%plot apparent radai of the three sprockets
% figure(2)
% hold on
% plot(ra_IK,'c')
% plot(ra_IC,'m')
% plot(ra_K,'b')
% plot(ra_C,'r')
% legend('Idler, Crank Side','Idler, Cassette Side','Crank','Cassette');
% title("Apparent Radius' of Sprockets");
% ylabel("Apparent Radius, [in]")
% hold off

xx(w,1)= max(GR_kc)- min(GR_kc);
```

```

xx(w,2) = N_i;
end
%plot change in Gear ratio
figure (2)
plot(xx(:,2),xx(:,1))
xticks(10:1:25)
grid on
xlabel('Idler Tooth Count')
ylabel('Change in Gear Ratio')
title('Static Height, Gear Ratio Variation')

```

Main_Sag:

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%// K = CRANK  I = IDLER  C = CASSETTE //
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

N_k = 32; %Number of teeth on crank
N_i = 11; %Number of teeth on idler
N_c = 50; %Number of teeth on cassette
P = 0.5; %Pitch
dtheta_init = 0.001; %Change in angle to fine initial position of
sprockets
TOL = 0.001; %Tolerance for finding initial sprocket positions
dtheta = 0.1; %Change in angle
Crank_OS = [0,0];
Idler_OS = [-28.65, 189.88]/25.4; %Coordinates of idler axle (offset)
Cassette_OS = [-437.39, 75.75]/25.4; %Coordinates of cassette axle
(offset)
steps = 45/dtheta; %Number of steps

%Move idler so chainline passes through same position
Idler_OS = MoveIdler(Idler_OS,N_i);

%initialize arrays
KI_Length = ones(steps,3);
IC_Length = ones(steps,2);
ra_IK = zeros(steps,1);
ra_K = zeros(steps,1);
ra_IC = zeros(steps,1);
ra_C = zeros(steps,1);

[Crank,r_k] = Create_Circle(P,N_k); %Create initial array of points for
crank

[Idler,r_i] = Create_Circle(P,N_i); %Create initial array of points for
idler

[Cassette,r_c] = Create_Circle(P,N_c); %Create initial array of points for
cassette

GR_ki = r_k/r_i; %crank-idler gear ratio
GR_kc = r_k/r_c; %crank-cassette gear ratio

%Adjust idler points with offset

```

```
for i = 1:N_i
    Idler(i,1) = Idler(i,1) + Idler_OS(1);
    Idler(i,2) = Idler(i,2) + Idler_OS(2);
end

%Adjust cassette points with offset
for i = 1:N_c
    Cassette(i,1) = Cassette(i,1) + Cassette_OS(1);
    Cassette(i,2) = Cassette(i,2) + Cassette_OS(2);
end

[Idler,idler_theta] =
Init_Position_KI(Crank,Idler,dtheta_init,N_k,N_i,TOL,r_i,GR_ki,Idler_OS);
[Cassette, theta_cassette] =
Init_Position_IC(Idler,Cassette,dtheta_init,N_i,N_c,TOL,r_c,GR_kc,Idler_OS
,Cassette_OS);

%Rotate crank full rotation in steps of dtheta

for i = 1:steps

T1 = FindTangent_KI(Crank,Idler,N_k,N_i,Idler_OS); %Calculate the tangent
line between the crank and the idler

T2 = FindTangent_IC(Idler,Cassette,N_i,N_c,Idler_OS,Cassette_OS);
%Calculate the tangent line between the idler and the cassette

%Plot points
figure(1)
drawnow
plot(Crank(1:N_k,1),Crank(1:N_k,2),'r.',Idler(1:N_i,1),Idler(1:N_i,2),'b.'
,Cassette(1:N_c,1),Cassette(1:N_c,2),'m.',T1(1:2,1),T1(1:2,2),T2(1:2,1),T2
(1:2,2));
axis equal
grid on

KI_Length(i,1) = dtheta*i; %Turn angle of crank
KI_Length(i,2) = sqrt((T1(2,1)-T1(1,1))^2 + (T1(2,2)-T1(1,2))^2);
%Calculate length of KI tangent line
KI_Length(i,3) = atan((T1(2,2)-T1(1,2))/(T1(2,1)-T1(1,1)));

IC_Length(i,1) = dtheta*i; %Turn angle of crank
IC_Length(i,2) = sqrt((T2(2,1)-T2(1,1))^2 + (T2(2,2)-T2(1,2))^2);
%Calculate length of IC tangent line

%calculate live gear ratios
[GR_ki(i), ra_IK(i), ra_K(i)] = SpinRatio(T1,Idler_OS, Crank_OS); %crank-
idler gear ratio
[GR_ic(i), ra_IC(i), ra_C(i)] = SpinRatio(T2,Idler_OS, Cassette_OS);
%crank-cassette gear ratio
GR_kc(i) = GR_ki(i)/GR_ic(i);

if i~=1
Crank = rotate(Crank,dtheta,N_k,r_k,1); %Rotate crank
```

```

Idler = rotate(Idler,dtheta,N_i,r_i,GR_ki); %Rotate idler relative to
crank

Cassette = rotate(Cassette,dtheta,N_c,r_c,GR_kc); %Rotate cassette
ralative to crank
end
end

% %Plot change in length of tangent lines
% figure(2)
%
plot(KI_Length(1:steps,1),KI_Length(1:steps,2),IC_Length(1:steps,1),IC_Len
gth(1:steps,2));
% legend('Crank/Idler','Idler/Cassette','Location','east');
%
% figure(3)
% plot(KI_Length(1:steps,1),(180/pi)*KI_Length(1:steps,3));

%plot apparent radai of the three sprockets
% figure(2)
% hold on
% plot(ra_IK,'c')
% plot(ra_IC,'m')
% plot(ra_K,'b')
% plot(ra_C,'r')
% legend('Idler, Crank Side','Idler, Cassette Side','Crank','Cassette');
% title("Apparent Radius' of Sprockets");
% ylabel("Apparent Radius, [in]")
% hold off
figure(2)
plot(GR_kc)
title('Gear Ratio Variation');
ylabel('Overall Gear Ratio');

```

Main_Sag_XYMove:

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%// K = CRANK   I = IDLER   C = CASSETTE //
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

N_k = 32; %Number of teeth on crank
N_i = 11; %Number of teeth on idler
N_c = 50; %Number of teeth on cassette
P = 0.5; %Pitch
dtheta_init = 0.001; %Change in angle to fine initial position of
sprockets
TOL = 0.001; %Tolerance for finding initial sprocket positions
dtheta = 0.1; %Change in angle
Crank_OS = [0,0];
Idler_OS = [-28.65, 189.88]/25.4; %Coordinates of idler axle (offset)
Cassette_OS = [-437.39, 75.75]/25.4; %Coordinates of cassette axle (offset)
steps = 20/dtheta; %Number of steps
xy_Step = 0.05; %step size for moving in xy [inches]

```

```
distXY = .5; %distance to travel from current position in +x,-x,+y,-y
directions
xy_Steps = 2*distXY/xy_Step;

%Move idler so chainline passes through same position
Idler_OS = MoveIdler(Idler_OS,N_i);

for n=0:xy_Steps%define position range array
    xy(n+1,[1 2]) = Idler_OS -[distXY distXY] + n*[xy_Step xy_Step];
end

for n=0:xy_Steps %loop through y
    Idler_OS(1,2) = xy(n+1,2);

    for m=0:xy_Steps %loop through x
        Idler_OS(1,1) = xy(m+1,1);

%initialize arrays
KI_Length = ones(steps,3);
IC_Length = ones(steps,2);
ra_IK = zeros(steps,1);
ra_K = zeros(steps,1);
ra_IC = zeros(steps,1);
ra_C = zeros(steps,1);

[Crank,r_k] = Create_Circle(P,N_k); %Create initial array of points for
crank

[Idler,r_i] = Create_Circle(P,N_i); %Create initial array of points for
idler

[Cassette,r_c] = Create_Circle(P,N_c); %Create intial array of points for
cassette

GR_ki = r_k/r_i; %crank-idler gear ratio
GR_kc = r_k/r_c; %crank-cassette gear ratio

%Adjust idler points with offset
for i = 1:N_i
    Idler(i,1) = Idler(i,1) + Idler_OS(1);
    Idler(i,2) = Idler(i,2) + Idler_OS(2);
end

%Adjust cassette points with offset
for i = 1:N_c
    Cassette(i,1) = Cassette(i,1) + Cassette_OS(1);
    Cassette(i,2) = Cassette(i,2) + Cassette_OS(2);
end

[Idler,idler_theta] =
Init_Position_KI(Crank,Idler,dtheta_init,N_k,N_i,TOL,r_i,GR_ki,Idler_OS);
[Cassette, theta_cassette] =
Init_Position_IC(Idler,Cassette,dtheta_init,N_i,N_c,TOL,r_c,GR_kc,Idler_OS
,Cassette_OS);

%Rotate crank full rotation in steps of dtheta
```

```

for i = 1:steps

T1 = FindTangent_KI(Crank,Idler,N_k,N_i,Idler_OS); %Calculate the tangent
line between the crank and the idler

T2 = FindTangent_IC(Idler,Cassette,N_i,N_c,Idler_OS,Cassette_OS);
%Calculate the tangent line between the idler and the cassette

%Plot points
figure(1)
%drawnow
plot(Crank(1:N_k,1),Crank(1:N_k,2),'r.',Idler(1:N_i,1),Idler(1:N_i,2),'b.'
,Cassette(1:N_c,1),Cassette(1:N_c,2),'m.',T1(1:2,1),T1(1:2,2),T2(1:2,1),T2
(1:2,2));
axis equal
grid on

KI_Length(i,1) = dtheta*i; %Turn angle of crank
KI_Length(i,2) = sqrt((T1(2,1)-T1(1,1))^2 + (T1(2,2)-T1(1,2))^2);
%Calculate length of KI tangent line
KI_Length(i,3) = atan((T1(2,2)-T1(1,2))/(T1(2,1)-T1(1,1)));

IC_Length(i,1) = dtheta*i; %Turn angle of crank
IC_Length(i,2) = sqrt((T2(2,1)-T2(1,1))^2 + (T2(2,2)-T2(1,2))^2);
%Calculate length of IC tangent line

%calculate live gear ratios
[GR_ki(i), ra_IK(i), ra_K(i)] = SpinRatio(T1,Idler_OS, Crank_OS); %crank-
idler gear ratio
[GR_ic(i), ra_IC(i), ra_C(i)] = SpinRatio(T2,Idler_OS, Cassette_OS);
%crank-cassette gear ratio
GR_kc(i) = GR_ki(i)/GR_ic(i);

if i~=1
Crank = rotate(Crank,dtheta,N_k,r_k,1); %Rotate crank

Idler = rotate(Idler,dtheta,N_i,r_i,GR_ki); %Rotate idler relative to
crank

Cassette = rotate(Cassette,dtheta,N_c,r_c,GR_kc); %Rotate cassette
relative to crank
end
end

% %Plot change in length of tangent lines
% figure(2)
%
plot(KI_Length(1:steps,1),KI_Length(1:steps,2),IC_Length(1:steps,1),IC_Len
gth(1:steps,2));
% legend('Crank/Idler','Idler/Cassette','Location','east');
%
% figure(3)

```

```
% plot(KI_Length(1:steps,1), (180/pi)*KI_Length(1:steps,3));

%plot apparent radai of the three sprockets
% figure(2)
% hold on
% plot(ra_IK, 'c')
% plot(ra_IC, 'm')
% plot(ra_K, 'b')
% plot(ra_C, 'r')
% legend('Idler, Crank Side','Idler, Cassette Side','Crank','Cassette');
% title("Apparent Radius' of Sprockets");
% ylabel("Apparent Radius, [in]")
% hold off

GR(n+1,m+1) = max(GR_kc) - min(GR_kc);

    end
end
```

Main_Sag_tSweep:

```
%////////////////////////////////////////
%// K = CRANK  I = IDLER  C = CASSETTE //
%////////////////////////////////////////

N_k = 32; %Number of teeth on crank
N_i = 16; %Number of teeth on idler
N_c = 50; %Number of teeth on cassette
P = 0.5; %Pitch
dtheta_init = 0.001; %Change in angle to fine initial position of
sprockets
TOL = 0.001; %Tolerance for finding initial sprocket positions
dtheta = 0.1; %Change in angle
Crank_OS = [0,0];
Idler_OS_old = [-28.65, 189.88]/25.4; %Coordinates of idler axle (offset);
%Coordinates of idler axle (offset)
Cassette_OS = [-437.39, 75.75]/25.4; %Coordinates of cassette axle (offset)
steps = 30/dtheta; %Number of steps

%Move idler so chainline passes through same position
Idler_OS = MoveIdler(Idler_OS_old,N_i);

w=0;
for N_i = 10:25
%Move idler so chainline passes through same position
Idler_OS = MoveIdler(Idler_OS_old,N_i);
    w = w+1;
%initialize arrays
KI_Length = ones(steps,3);
IC_Length = ones(steps,2);
ra_IK = zeros(steps,1);
ra_K = zeros(steps,1);
ra_IC = zeros(steps,1);
ra_C = zeros(steps,1);
```

```

[Crank,r_k] = Create_Circle(P,N_k); %Create initial array of points for
crank

[Idler,r_i] = Create_Circle(P,N_i); %Create initial array of points for
idler

[Cassette,r_c] = Create_Circle(P,N_c); %Create initial array of points for
cassette

GR_ki = r_k/r_i; %crank-idler gear ratio
GR_kc = r_k/r_c; %crank-cassette gear ratio

%Adjust idler points with offset
for i = 1:N_i
    Idler(i,1) = Idler(i,1) + Idler_OS(1);
    Idler(i,2) = Idler(i,2) + Idler_OS(2);
end

%Adjust cassette points with offset
for i = 1:N_c
    Cassette(i,1) = Cassette(i,1) + Cassette_OS(1);
    Cassette(i,2) = Cassette(i,2) + Cassette_OS(2);
end

[Idler,idler_theta] =
Init_Position_KI(Crank,Idler,dtheta_init,N_k,N_i,TOL,r_i,GR_ki,Idler_OS);
[Cassette, theta_cassette] =
Init_Position_IC(Idler,Cassette,dtheta_init,N_i,N_c,TOL,r_c,GR_kc,Idler_OS
,Cassette_OS);

%Rotate crank full rotation in steps of dtheta

for i = 1:steps

T1 = FindTangent_KI(Crank,Idler,N_k,N_i,Idler_OS); %Calculate the tangent
line between the crank and the idler

T2 = FindTangent_IC(Idler,Cassette,N_i,N_c,Idler_OS,Cassette_OS);
%Calculate the tangent line between the idler and the cassette

%Plot points
figure(1)
drawnow
plot(Crank(1:N_k,1),Crank(1:N_k,2),'r.',Idler(1:N_i,1),Idler(1:N_i,2),'b.'
,Cassette(1:N_c,1),Cassette(1:N_c,2),'m.',T1(1:2,1),T1(1:2,2),T2(1:2,1),T2
(1:2,2));
axis equal
grid on

KI_Length(i,1) = dtheta*i; %Turn angle of crank
KI_Length(i,2) = sqrt((T1(2,1)-T1(1,1))^2 + (T1(2,2)-T1(1,2))^2);
%Calculate length of KI tangent line
KI_Length(i,3) = atan((T1(2,2)-T1(1,2))/(T1(2,1)-T1(1,1)));

```

```
IC_Length(i,1) = dtheta*i; %Turn angle of crank
IC_Length(i,2) = sqrt((T2(2,1)-T2(1,1))^2 + (T2(2,2)-T2(1,2))^2);
%Calculate length of IC tangent line

%calculate live gear ratios
[GR_ki(i), ra_IK(i), ra_K(i)] = SpinRatio(T1,Idler_OS, Crank_OS); %crank-
idler gear ratio
[GR_ic(i), ra_IC(i), ra_C(i)] = SpinRatio(T2,Idler_OS, Cassette_OS);
%crank-cassette gear ratio
GR_kc(i) = GR_ki(i)/GR_ic(i);

if i~=1
Crank = rotate(Crank,dtheta,N_k,r_k,1); %Rotate crank

Idler = rotate(Idler,dtheta,N_i,r_i,GR_ki); %Rotate idler relative to
crank

Cassette = rotate(Cassette,dtheta,N_c,r_c,GR_kc); %Rotate cassette
ralative to crank
end
end

% %Plot change in length of tangent lines
% figure(2)
%
plot(KI_Length(1:steps,1),KI_Length(1:steps,2),IC_Length(1:steps,1),IC_Len
gth(1:steps,2));
% legend('Crank/Idler','Idler/Cassette','Location','east');
%
% figure(3)
% plot(KI_Length(1:steps,1),(180/pi)*KI_Length(1:steps,3));

%plot apparent radai of the three sprockets
% figure(2)
% hold on
% plot(ra_IK,'c')
% plot(ra_IC,'m')
% plot(ra_K,'b')
% plot(ra_C,'r')
% legend('Idler, Crank Side','Idler, Cassette Side','Crank','Cassette');
% title("Apparent Radius' of Sprockets");
% ylabel("Apparent Radius, [in]")
% hold off

xx(w,1)= max(GR_kc)- min(GR_kc); %xx is change in Gear Ratio array
xx(w,2) = N_i;
end
%plot change in Gear Ratio over tooth sweep
figure (2)
plot(xx(:,2),xx(:,1))
xticks(10:1:25)
grid on
xlabel('Idler Tooth Count')
ylabel('Change in Gear Ratio')
title('25% Sag Height, Gear Ratio Variation')
```

Load_Cell_Calc:

```

DT_GR_max = 0.6408; %Best case gear ratios
DT_GR_min = 0.6378;
%DT_GR_max = 0.6645; %Worst case gear ratios
%DT_GR_min = 0.6169;
Rear_Pulley_R = 40; %Cassette pulley radius
Front_Pulley_R = 100; %Crank pulley radius
K_R = 69.04; %Crank radius
C_R = 105.38; %Cassette radius
Load_Cell_diff = ones(50,3); %Difference in load cell tensions

%Run loop for various masses (kg)
for Weight = 50:-1:1
j = 51 - Weight; %Index
%Gear reduction calculations from front pulley to scale
Load_Cell_min = Weight*(Front_Pulley_R/K_R)*DT_GR_min*(C_R/Rear_Pulley_R);
Load_Cell_max = Weight*(Front_Pulley_R/K_R)*DT_GR_max*(C_R/Rear_Pulley_R);

Load_Cell_diff(j,1) = Weight; %Mass (kg)
Load_Cell_diff(j,2) = Load_Cell_max - Load_Cell_min; %Change in mass
Load_Cell_diff(j,3) = Weight*2.205; %Mass (lbm)
end

```

Appendix B – Simulation Data

B.1 – Single Position Results

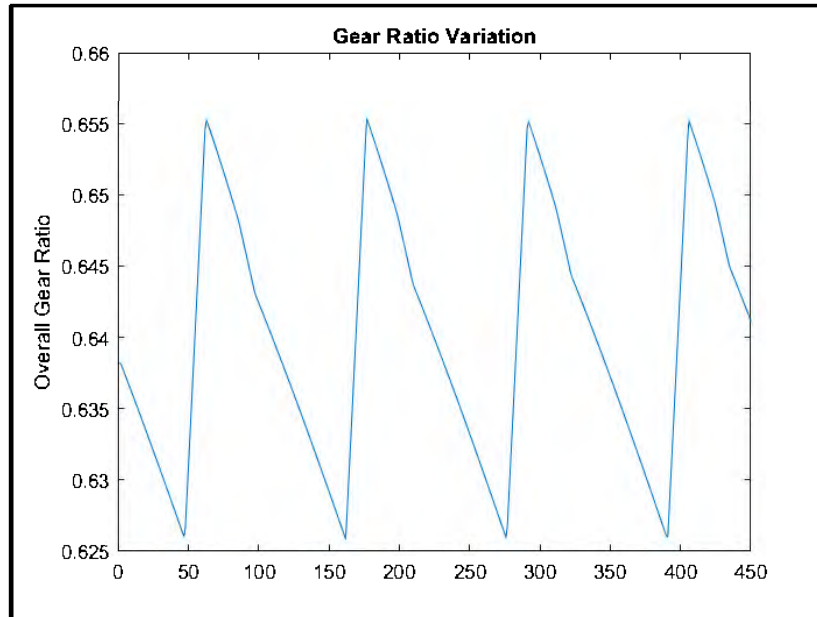


Figure B.1 10 Tooth 25% Sag

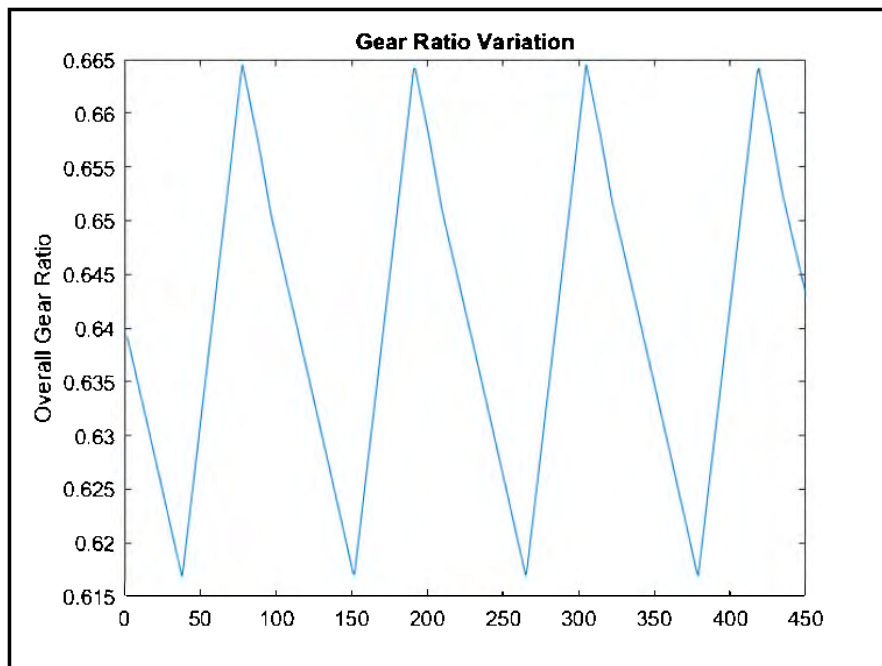


Figure B.2 11 Tooth 25% Sag

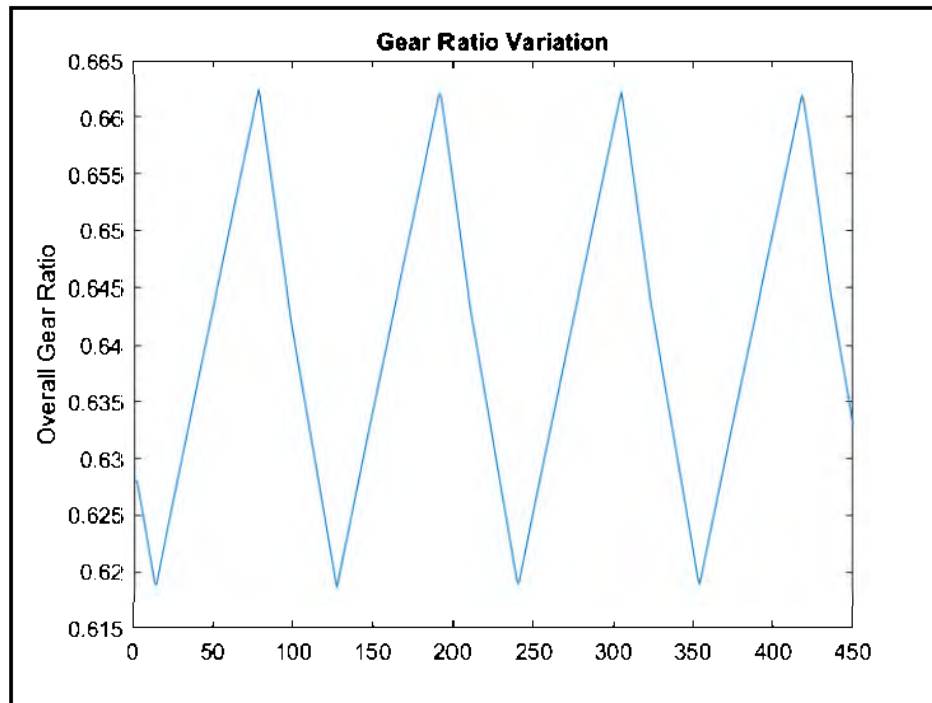


Figure B.3 12 Tooth 25% Sag

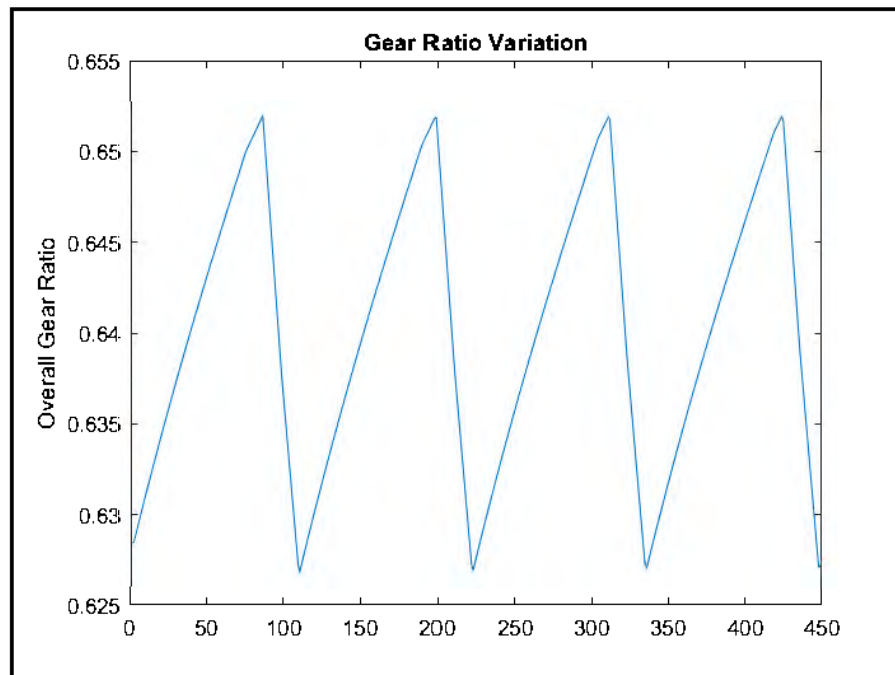


Figure B.4 13 Tooth 25% Sag

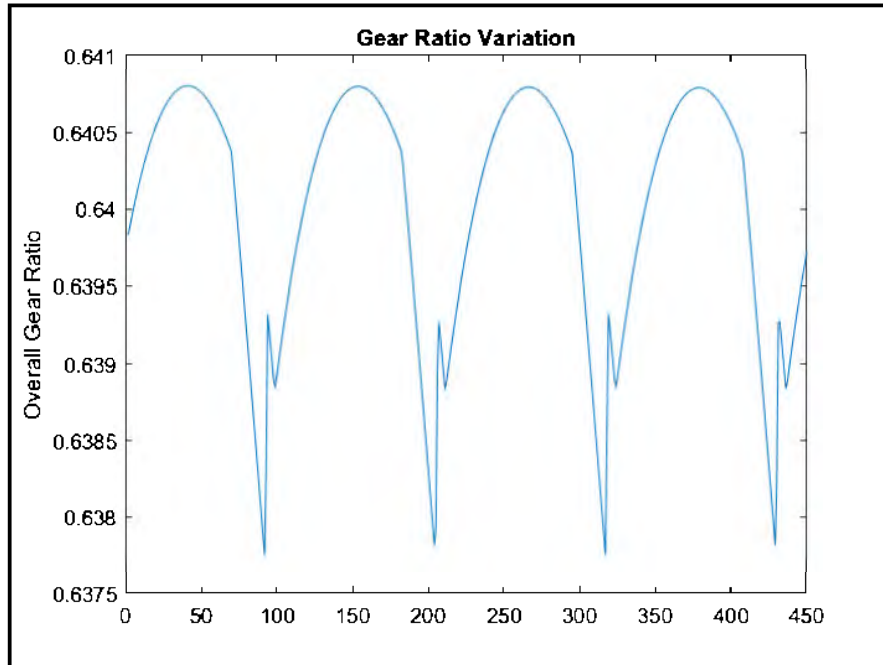


Figure B.5 14 Tooth 25% Sag

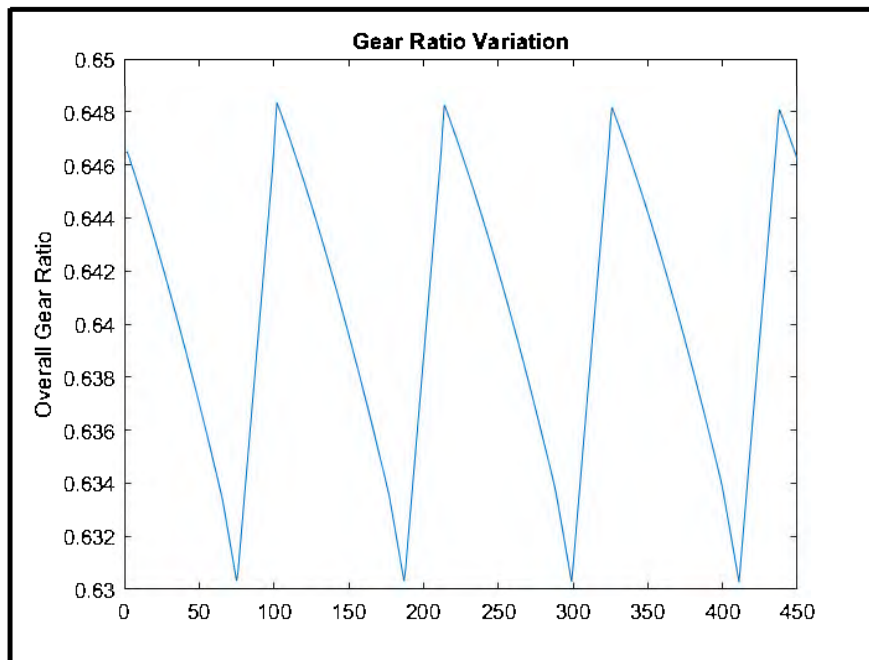


Figure B.6 15 Tooth 25% Sag

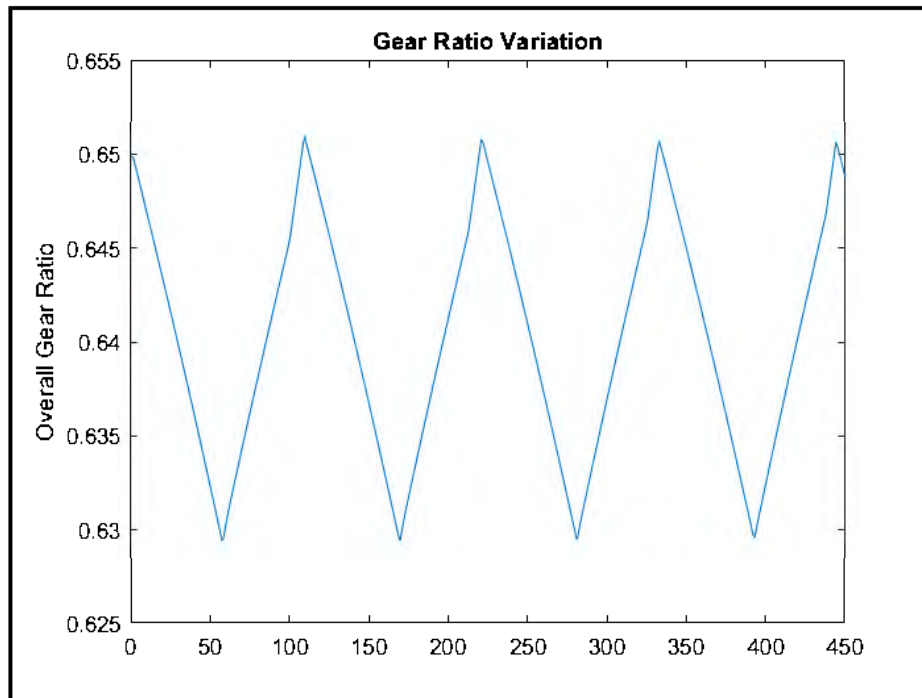


Figure B.7 16 Tooth 25% Sag

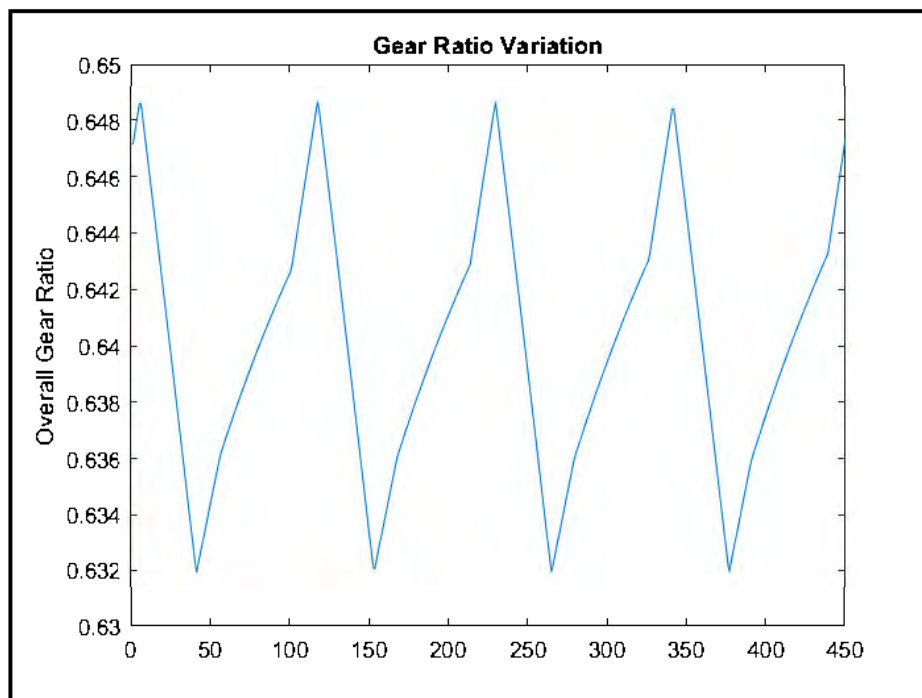


Figure B.8 17 Tooth 25% Sag

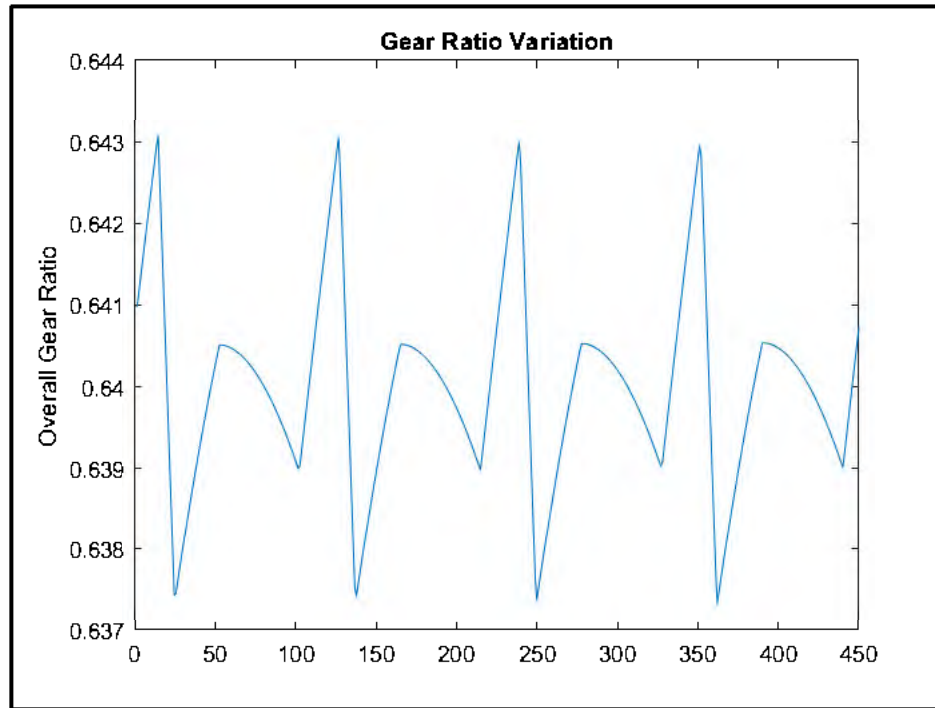


Figure B.9 18 Tooth 25% Sag

		X Position																					
		0																					
Y Position		-1.34929	-1.29929	-1.24929	-1.19929	-1.14929	-1.09929	-1.04929	-0.99929	-0.94929	-0.89929	-0.84929	-0.79929	-0.74929	-0.69929	-0.64929	-0.59929	-0.54929	-0.49929	-0.44929	-0.39929	-0.34929	Relative Y Position
7.254255211	0.028189	0.028189	0.029797	0.031593	0.033097	0.034479	0.035343	0.035585	0.036354	0.037696	0.039096	0.040516	0.041537	0.042837	0.122156	0.044977	0.044551	0.045121	0.045782	0.046736	0.047731	-0.5	
7.304255211	0.028585	0.030108	0.031192	0.033372	0.034963	0.036287	0.037081	0.037211	0.038511	0.039769	0.040806	0.042176	0.04343	0.044381	0.045533	0.045514	0.046324	0.045656	0.04696	0.048023	0.049067	-0.45	
7.354255211	0.030578	0.032119	0.033944	0.035711	0.037246	0.038163	0.038724	0.039011	0.039897	0.041311	0.042783	0.04396	0.045197	0.04622	0.046915	0.047183	0.046774	0.047143	0.047761	0.048578	0.049484	-0.4	
7.404255211	0.032683	0.034173	0.035791	0.037482	0.039308	0.039863	0.040262	0.040647	0.042123	0.043143	0.044366	0.045783	0.04714	0.048051	0.048467	0.048765	0.04854	0.048703	0.049339	0.050506	0.051628	-0.35	
7.454255211	0.034725	0.036173	0.037819	0.039386	0.040854	0.041563	0.042013	0.043354	0.043973	0.045209	0.046293	0.047517	0.04874	0.050109	0.050364	0.050517	0.050224	0.050776	0.051602	0.052519	0.053282	-0.3	
7.504255211	0.034719	0.036526	0.038102	0.039844	0.040577	0.041124	0.041711	0.04275	0.04438	0.045374	0.046899	0.047851	0.049609	0.050448	0.05072	0.051043	0.050927	0.051339	0.052479	0.053527	0.054286	-0.25	
7.554255211	0.035071	0.037148	0.037586	0.039327	0.039698	0.040195	0.040837	0.042221	0.043241	0.04468	0.045725	0.0475	0.048537	0.048981	0.049077	0.049344	0.049538	0.050415	0.051169	0.051865	0.052468	-0.2	
7.604255211	0.03464	0.035941	0.037758	0.039184	0.038621	0.039296	0.040026	0.041194	0.042615	0.043707	0.044935	0.046018	0.047396	0.047626	0.048015	0.047903	0.048083	0.049153	0.050042	0.050768	0.05175	-0.15	
7.654255211	0.036544	0.037842	0.038932	0.039629	0.039831	0.039534	0.040324	0.042072	0.042937	0.043873	0.045252	0.045989	0.046683	0.046273	0.046245	0.045966	0.046962	0.047626	0.048972	0.04927	0.05051	-0.1	
7.704255211	0.038126	0.03948	0.040995	0.041204	0.041284	0.041291	0.042177	0.04319	0.044072	0.044809	0.046597	0.047797	0.047428	0.047516	0.047664	0.047333	0.048257	0.048825	0.049561	0.050161	0.050757	-0.05	
7.754255211	0.040261	0.041546	0.04237	0.042612	0.042693	0.042882	0.043797	0.044771	0.04553	0.046802	0.047655	0.048362	0.047657	0.048943	0.048336	0.048598	0.04907	0.050041	0.050821	0.051949	0.052385	0	
7.804255211	0.042093	0.043436	0.043747	0.043914	0.044179	0.044945	0.045581	0.046624	0.047443	0.048492	0.049062	0.049676	0.049617	0.049208	0.049004	0.048964	0.05082	0.051844	0.052243	0.053149	0.0536	0.05	
7.854255211	0.044099	0.045504	0.045756	0.04525	0.045555	0.046445	0.047506	0.048462	0.049363	0.050247	0.050685	0.051589	0.050891	0.050739	0.050322	0.051011	0.051406	0.051975	0.052486	0.052877	0.054837	0.1	
7.904255211	0.046106	0.0469	0.047147	0.047454	0.047545	0.048157	0.049262	0.050275	0.050789	0.052042	0.052491	0.053203	0.052471	0.052262	0.051905	0.052271	0.053019	0.053891	0.054716	0.055135	0.054641	0.15	
7.954255211	0.045444	0.045821	0.046507	0.046134	0.04723	0.048364	0.049482	0.050517	0.051665	0.052876	0.053642	0.053716	0.053318	0.05321	0.05327	0.054042	0.054802	0.055511	0.056413	0.056982	0.056552	0.2	
8.004255211	0.044546	0.044797	0.045158	0.045317	0.046359	0.047268	0.048634	0.049714	0.050816	0.051764	0.051997	0.052109	0.052009	0.051683	0.051999	0.052798	0.053653	0.054473	0.054886	0.054931	0.055108	0.25	
8.054255211	0.043401	0.043876	0.044028	0.044239	0.045463	0.046679	0.047723	0.048462	0.049296	0.050648	0.050765	0.050422	0.050086	0.050464	0.05072	0.051763	0.052377	0.052952	0.053369	0.053982	0.053278	0.3	
8.104255211	0.043424	0.043435	0.044057	0.044295	0.045004	0.045705	0.046995	0.047304	0.048921	0.049065	0.04896	0.048726	0.048803	0.048823	0.050507	0.051452	0.051884	0.052411	0.052138	0.051337	0.35		
8.154255211	0.044756	0.044579	0.044429	0.045157	0.046089	0.047402	0.047989	0.048822	0.049477	0.049208	0.049234	0.048828	0.048434	0.048488	0.048862	0.049104	0.050081	0.050401	0.05089	0.050367	0.049935	0.4	
8.204255211	0.046631	0.046103	0.045896	0.046603	0.047353	0.048341	0.049056	0.04983	0.050871	0.050644	0.049977	0.049475	0.049017	0.04985	0.050101	0.05115	0.051706	0.052204	0.050998	0.050799	0.050209	0.45	
8.254255211	0.04768	0.047647	0.048066	0.0483	0.049133	0.050117	0.050497	0.051224	0.051282	0.050714	0.050124	0.050848	0.050666	0.051325	0.052185	0.052446	0.052676	0.053492	0.052618	0.052113	0.051556	0.5	
		-0.5	-0.45	-0.4	-0.35	-0.3	-0.25	-0.2	-0.15	-0.1	-0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	
		Relative X Position																					
DATA																							
Current Change in gear ratio:		0.047655																				*All Dimensions in inches	
Relative X:		0																					
Relative Y:		0																					
Optimal Change in gear ratio:		0.026477																					
Relative X:		-0.5																					
Relative Y:		-0.5																					
Worst Case Change in Gear ratio:		0.251836																					

		X Position																					
		-1.40484	-1.35484	-1.30484	-1.25484	-1.20484	-1.15484	-1.10484	-1.05484	-1.00484	-0.95484	-0.90484	-0.85484	-0.80484	-0.75484	-0.70484	-0.65484	-0.60484	-0.55484	-0.50484	-0.45484	-0.40484	
Y Position																						Relative Y Position	
7.198705296		0.039988	0.04058	0.041426	0.042225	0.04265	0.04313	0.042536	0.042223	0.041572	0.04179	0.042107	0.042472	0.042832	0.043525	0.0431	0.042595	0.041451	0.040944	0.040435	0.039942	0.040041	-0.5
7.248705296		0.041804	0.042506	0.043114	0.043916	0.044541	0.044062	0.043487	0.043477	0.042897	0.042833	0.043473	0.043916	0.044465	0.044973	0.044277	0.043741	0.04276	0.042032	0.041311	0.041268	0.041763	-0.45
7.298705296		0.043314	0.043961	0.045068	0.045969	0.045996	0.045277	0.044955	0.044401	0.044015	0.044745	0.045233	0.045626	0.045822	0.046146	0.045772	0.044794	0.044085	0.043299	0.042467	0.042622	0.04292	-0.4
7.348705296		0.044287	0.044933	0.046267	0.04722	0.047033	0.046756	0.04641	0.045744	0.045978	0.046054	0.046543	0.046979	0.047621	0.047596	0.046652	0.04603	0.044893	0.044523	0.043972	0.043994	0.043768	-0.35
7.398705296		0.043548	0.044425	0.045148	0.045839	0.045625	0.045295	0.044946	0.044664	0.045041	0.045574	0.046245	0.046533	0.047288	0.046312	0.045667	0.044862	0.044205	0.043412	0.042541	0.042929	0.042943	-0.3
7.448705296		0.042483	0.043009	0.044217	0.044441	0.044136	0.043967	0.043578	0.043088	0.04377	0.044507	0.045157	0.045636	0.045907	0.045077	0.044495	0.043744	0.042831	0.041892	0.04185	0.041583	0.041947	-0.25
7.498705296		0.041553	0.042592	0.043409	0.043334	0.043216	0.042934	0.042056	0.04185	0.042623	0.043376	0.0441	0.044728	0.044469	0.043679	0.042749	0.042092	0.231915	0.040717	0.040343	0.040853	0.040664	-0.2
7.548705296		0.040594	0.041548	0.042128	0.041973	0.041757	0.041369	0.041005	0.041046	0.041884	0.042391	0.042429	0.043192	0.042744	0.042017	0.041137	0.040584	0.040067	0.039093	0.039633	0.039782	0.03985	-0.15
7.598705296		0.041835	0.042683	0.042707	0.04214	0.041361	0.040915	0.040228	0.040007	0.040662	0.041122	0.041485	0.041685	0.040799	0.040274	0.039862	0.039061	0.038302	0.038116	0.038284	0.038735	0.038004	-0.1
7.648705296		0.043637	0.044009	0.043936	0.043191	0.042371	0.04195	0.041201	0.041441	0.04153	0.042048	0.042397	0.041497	0.040964	0.039813	0.038953	0.038327	0.037194	0.03679	0.037073	0.037207	0.036775	-0.05
7.698705296		0.045495	0.045729	0.044715	0.044474	0.043798	0.043093	0.042591	0.042878	0.043193	0.043474	0.043803	0.042961	0.041734	0.041279	0.039934	0.038954	0.038525	0.038334	0.038684	0.038559	0.037263	0
7.748705296		0.047219	0.046907	0.046428	0.045573	0.044618	0.044245	0.044137	0.044418	0.044512	0.044611	0.044998	0.044052	0.043147	0.042238	0.041278	0.040604	0.039828	0.039753	0.040144	0.039779	0.03863	0.05
7.798705296		0.047951	0.047656	0.047208	0.046655	0.046343	0.045385	0.04555	0.045668	0.046047	0.046455	0.045927	0.04467	0.04413	0.043371	0.042264	0.041753	0.041169	0.04118	0.041097	0.040472	0.039349	0.1
7.848705296		0.046707	0.046227	0.045462	0.045167	0.044842	0.044477	0.044983	0.045148	0.045843	0.045894	0.044882	0.044284	0.04341	0.042759	0.04136	0.040483	0.040218	0.040271	0.040132	0.039441	0.038046	0.15
7.898705296		0.045187	0.044674	0.044101	0.043695	0.043168	0.043417	0.043934	0.044329	0.044787	0.044558	0.043716	0.042938	0.04192	0.040836	0.040323	0.039242	0.039386	0.039293	0.039294	0.038176	0.036555	0.2
7.948705296		0.043927	0.043089	0.042695	0.041976	0.041853	0.042226	0.042773	0.04307	0.043563	0.042883	0.042094	0.041303	0.040617	0.039817	0.038584	0.038318	0.03814	0.038288	0.037843	0.036646	0.035331	0.25
7.998705296		0.042327	0.041748	0.041315	0.040596	0.040763	0.041038	0.041266	0.041629	0.041991	0.041049	0.040635	0.039944	0.038811	0.038045	0.037355	0.037251	0.037323	0.037373	0.036105	0.034865	0.033993	0.3
8.048705296		0.041538	0.040767	0.040033	0.039587	0.039682	0.039776	0.040309	0.040516	0.040561	0.039362	0.039266	0.038403	0.037263	0.03667	0.03581	0.036275	0.03583	0.035839	0.034643	0.033607	0.032358	0.35
0	8.098705296	0.042591	0.041794	0.040934	0.039869	0.039665	0.040012	0.040062	0.039951	0.039419	0.038265	0.037217	0.036434	0.036047	0.034816	0.034749	0.034552	0.034961	0.03422	0.032954	0.032139	0.030912	0.4
	8.148705296	0.043455	0.042976	0.04203	0.04113	0.041266	0.04134	0.041508	0.041085	0.040232	0.039283	0.038283	0.037105	0.035911	0.035	0.034906	0.03471	0.034398	0.033304	0.031966	0.03067	0.029532	0.45
	8.198705296	0.044408	0.043606	0.043039	0.042435	0.042441	0.042528	0.042954	0.042451	0.041418	0.040408	0.038984	0.038383	0.037146	0.036173	0.036315	0.03615	0.03567	0.034364	0.033387	0.031822	0.030664	0.5
		-0.5	-0.45	-0.4	-0.35	-0.3	-0.25	-0.2	-0.15	-0.1	-0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	
		Relative X Position																					
DATA																							
Current Change in gear ratio:		0.043803																				*All Dimensions in Inches	
Relative X:		0																					
Relative Y:		0																					
Optimal Change in gear ratio:		0.029532																					
Relative X:		0.5																					
Relative Y:		0.45																					
Worst Case Change in Gear ratio:		0.231915																					

Figure B.11 12 Tooth 25% Sag Starting Position

		X Position																					Relative Y Position
Y Position		-1.4605	-1.4105	-1.3605	-1.3105	-1.2605	-1.2105	-1.1605	-1.1105	-1.0605	-1.0105	-0.9605	-0.9105	-0.8605	-0.8105	-0.7605	-0.7105	-0.6605	-0.6105	-0.5605	-0.5105	-0.4605	
7.143042174		0.036717	0.037021	0.037077	0.037025	0.036273	0.035473	0.034483	0.033265	0.032239	0.031103	0.030185	0.029997	0.029976	0.029217	0.027848	0.026432	0.025162	0.023878	0.02235	0.0207	0.019698	-0.5
7.193042174		0.03847	0.038484	0.038488	0.038793	0.037725	0.036391	0.035522	0.034415	0.033463	0.032245	0.031584	0.031372	0.031258	0.030086	0.028981	0.027491	0.026218	0.024648	0.023447	0.022171	0.021164	-0.45
7.243042174		0.038104	0.03868	0.038873	0.039047	0.037914	0.037308	0.036211	0.233744	0.034389	0.033288	0.032817	0.032792	0.032485	0.031024	0.029695	0.028236	0.026894	0.025264	0.023828	0.022111	0.02133	-0.4
7.293042174		0.037229	0.037593	0.03788	0.03732	0.036552	0.035687	0.034793	0.033846	0.032753	0.031963	0.031974	0.031755	0.030994	0.029535	0.028327	0.026795	0.025482	0.023962	0.022456	0.02077	0.211666	-0.35
7.343042174		0.036061	0.03649	0.036423	0.035619	0.034751	0.034244	0.033372	0.032261	0.031306	0.030787	0.030654	0.030755	0.029414	0.028229	0.026834	0.025586	0.023829	0.022462	0.021044	0.019747	0.019212	-0.3
7.393042174		0.034711	0.035416	0.034939	0.034218	0.033527	0.032556	0.031678	0.030816	0.029911	0.029617	0.029592	0.029118	0.027839	0.026693	0.02531	0.023796	0.022538	0.021234	0.019686	0.018811	0.017876	-0.25
7.443042174		0.033712	0.034055	0.033986	0.032771	0.031922	0.031101	0.030192	0.029208	0.02823	0.028088	0.028417	0.027571	0.026402	0.02504	0.023716	0.022475	0.02116	0.019547	0.018216	0.017618	0.016505	-0.2
7.493042174		0.032732	0.032841	0.032242	0.031405	0.030578	0.02958	0.028544	0.027745	0.026877	0.026971	0.027071	0.026094	0.02498	0.023696	0.022293	0.021031	0.019702	0.01826	0.016967	0.016538	0.015044	-0.15
7.543042174		0.032784	0.032299	0.03102	0.029851	0.028952	0.028119	0.027371	0.026148	0.025773	0.025764	0.025566	0.024198	0.023082	0.021952	0.020715	0.019383	0.018163	0.016855	0.015564	0.01505	0.013578	-0.1
7.593042174		0.034285	0.033383	0.032176	0.030714	0.02956	0.028222	0.027155	0.025909	0.025248	0.025068	0.024054	0.022679	0.021666	0.020244	0.019312	0.018062	0.016751	0.015271	0.014581	0.013436	0.012022	-0.05
7.643042174		0.035595	0.034308	0.033382	0.032068	0.030776	0.029453	0.028221	0.027162	0.02671	0.026395	0.025135	0.023711	0.022304	0.020751	0.019277	0.017848	0.016575	0.014797	0.014014	0.012473	0.01067	0
7.693042174		0.035338	0.034388	0.033356	0.032331	0.031303	0.030109	0.028899	0.028144	0.220872	0.027246	0.025949	0.024403	0.022906	0.021501	0.019998	0.018408	0.0168	0.015401	0.014689	0.012929	0.011115	0.05
7.743042174		0.033898	0.032903	0.032009	0.030924	0.029506	0.028653	0.027495	0.02715	0.026906	0.025962	0.024467	0.022957	0.021513	0.020102	0.018677	0.017045	0.015386	0.014228	0.013244	0.011574	0.009817	0.1
7.793042174		0.032328	0.031431	0.03035	0.029345	0.028362	0.027259	0.026208	0.025858	0.025641	0.024234	0.023001	0.021551	0.020147	0.01867	0.017173	0.015652	0.014006	0.013248	0.011747	0.010018	0.008308	0.15
7.843042174		0.030831	0.029801	0.028868	0.027874	0.02681	0.025421	0.025057	0.024786	0.024081	0.02283	0.021502	0.020085	0.018749	0.017234	0.015669	0.014233	0.012807	0.012024	0.010336	0.008636	0.006932	0.2
7.893042174		0.029206	0.02831	0.027258	0.026027	0.586429	0.024026	0.02363	0.023718	0.022612	0.021274	0.019956	0.018677	0.017188	0.01576	0.014429	0.012794	0.011852	0.01058	0.008961	0.007234	0.005547	0.25
7.943042174		0.027751	0.026682	0.025591	0.024509	0.023539	0.022667	0.022456	0.022174	0.020935	0.019863	0.018343	0.016999	0.015718	0.014379	0.012884	0.011449	0.010589	0.009127	0.00748	0.005824	0.004152	0.3
7.993042174		0.026166	0.025173	0.024376	0.023229	0.022049	0.021392	0.021074	0.020522	0.01942	0.018105	0.016895	0.015605	0.014341	0.012661	0.011366	0.010031	0.009178	0.00761	0.006008	0.004417	0.003307	0.35
8.043042174		0.026054	0.024614	0.023147	0.021952	0.020673	0.0198	0.0199	0.019117	0.017679	0.016559	0.015591	0.014117	0.012525	0.011342	0.009958	0.008871	0.007689	0.006089	0.004608	0.003188	0.003898	0.4
8.093042174		0.027141	0.025748	0.024199	0.023016	0.021751	0.021048	0.02042	0.01888	0.017513	0.015918	0.014378	0.012829	0.011295	0.009974	0.008523	0.007691	0.006141	0.005189	0.004592	0.00402	0.004732	0.45
8.143042174		0.027239	0.025939	0.024718	0.023411	0.022415	0.022078	0.020951	0.019559	0.018065	0.016626	0.015065	0.013549	0.011925	0.010329	0.008723	0.007878	0.006824	0.005899	0.005113	0.004324	0.004858	0.5
		Relative X Position																					
		DATA																					
Current Change in gear ratio:		0.025135																				*All Dimensions in Inches	
Relative X:		0																					
Relative Y:		0																					
Optimal Change in gear ratio:		0.003188																					
Relative X:		0.45																					
Relative Y:		0.4																					
Worst Case Change in Gear ratio:		0.586429																					

Figure B.12 13 Tooth 25% Sag Starting Position

		X Position																					
																						0	
Y Position		-1.51625	-1.46625	-1.41625	-1.36625	-1.31625	-1.26625	-1.21625	-1.16625	-1.11625	-1.06625	-1.01625	-0.96625	-0.91625	-0.86625	-0.81625	-0.76625	-0.71625	-0.66625	-0.61625	-0.56625	-0.51625	Relative Y Position
	7.087291	0.02451	0.023653	0.023017	0.022243	0.020524	0.019012	0.017467	0.015867	0.014148	0.012469	0.010744	0.009368	0.008407	0.007406	0.006435	0.00551	0.004661	0.004451	0.005413	0.006335	0.007235	-0.5
	7.137291	0.023706	0.023114	0.023124	0.021818	0.020512	0.019025	0.017589	0.016091	0.014441	0.012837	0.011095	0.010066	0.008812	0.007563	0.006385	0.00528	0.004205	0.003923	0.004638	0.005456	0.006495	-0.45
	7.187291	0.022119	0.022023	0.021657	0.020323	0.018947	0.017499	0.016038	0.014549	0.012961	0.011356	0.009888	0.009181	0.007703	0.006364	0.005092	0.00392	0.00278	0.002901	0.004168	0.00594	0.007449	-0.4
	7.237291	0.020894	0.020691	0.02011	0.018853	0.017413	0.015941	0.014523	0.013046	0.01147	0.009937	0.009069	0.008066	0.006482	0.005012	0.003698	0.002461	0.001927	0.003716	0.005546	0.007305	0.00845	-0.35
	7.287291	0.019611	0.019466	0.018552	0.01724	0.015907	0.014431	0.01294	0.01151	0.010036	0.008533	0.008147	0.006863	0.005392	0.003869	0.00238	0.00219	0.003349	0.005123	0.006955	0.00873	0.009548	-0.3
	7.337291	0.018434	0.018089	0.017038	0.015522	0.01411	0.012819	0.011488	0.010098	0.00847	0.007422	0.006984	0.005639	0.004209	0.002787	0.002463	0.003633	0.005055	0.006625	0.008435	0.009852	0.010933	-0.25
	7.387291	0.017235	0.016694	0.015379	0.014064	0.012642	0.011334	0.010021	0.008414	0.007044	0.006006	0.005457	0.004246	0.002976	0.002719	0.004074	0.005572	0.007022	0.00851	0.010019	0.011007	0.012317	-0.2
	7.437291	0.01604	0.014987	0.013667	0.012437	0.011012	0.00978	0.008455	0.007002	0.005454	0.004501	0.003754	0.002649	0.00297	0.004326	0.005705	0.007234	0.008819	0.010388	0.011999	0.012386	0.013773	-0.15
	7.487291	0.0147	0.013239	0.012135	0.010784	0.009621	0.008226	0.006823	0.005379	0.004001	0.002695	0.001803	0.002997	0.004385	0.005799	0.0073	0.00881	0.010397	0.012016	0.013286	0.013724	0.015293	-0.1
	7.537291	0.015622	0.014002	0.012145	0.010455	0.008718	0.007044	0.005615	0.004147	0.002673	0.002117	0.002759	0.004169	0.005671	0.007141	0.008658	0.010223	0.011835	0.013455	0.014403	0.015302	0.01685	-0.05
	7.587291	0.014695	0.01324	0.011699	0.010144	0.008517	0.006861	0.005189	0.003635	0.002709	0.002333	0.00305	0.004342	0.005815	0.007196	0.008618	0.010199	0.011586	0.013109	0.01364	0.01482	0.016297	0
	7.637291	0.01312	0.011643	0.010182	0.008626	0.007017	0.005397	0.003811	0.003147	0.002544	0.002143	0.002781	0.004229	0.005842	0.007285	0.008716	0.010273	0.011837	0.012988	0.013392	0.014399	0.015732	0.05
	7.687291	0.011563	0.010159	0.008664	0.007061	0.005533	0.004339	0.003813	0.003254	0.003149	0.003104	0.004052	0.005609	0.007165	0.008636	0.010099	0.011611	0.013044	0.013852	0.014461	0.015625	0.016802	0.1
	7.737291	0.010018	0.008577	0.007057	0.005618	0.00464	0.0042	0.003801	0.004225	0.004348	0.004427	0.005457	0.00701	0.008505	0.009991	0.011458	0.013018	0.014414	0.014837	0.015789	0.016863	0.018051	0.15
	7.787291	0.008375	0.006961	0.005609	0.00452	0.004258	0.004002	0.004755	0.005258	0.005468	0.005618	0.006879	0.008395	0.0099	0.011368	0.012957	0.01447	0.015453	0.015878	0.016973	0.018177	0.019451	0.2
	7.837291	0.006786	0.005386	0.003975	0.003851	0.004244	0.005057	0.005805	0.006197	0.006448	0.006799	0.008327	0.009833	0.011323	0.012872	0.014421	0.0159	0.016509	0.017111	0.018365	0.019643	0.020835	0.25
	7.887291	0.005303	0.003873	0.003488	0.004356	0.0052	0.00592	0.006624	0.006971	0.00736	0.008353	0.009858	0.011402	0.01296	0.014459	0.015951	0.017476	0.017805	0.018572	0.019835	0.021077	0.02223	0.3
	7.937291	0.004778	0.00407	0.004227	0.005024	0.005768	0.00646	0.007078	0.00756	0.008567	0.009956	0.011509	0.012997	0.014549	0.016038	0.017523	0.018669	0.019023	0.020046	0.021329	0.022549	0.023861	0.35
0	7.987291	0.005386	0.004502	0.004623	0.005334	0.005997	0.006613	0.007215	0.008775	0.010343	0.011801	0.013244	0.014721	0.016216	0.017743	0.019256	0.019918	0.020316	0.021486	0.022957	0.024199	0.025429	0.4
	8.037291	0.004407	0.003471	0.00389	0.004518	0.005131	0.005918	0.006898	0.008371	0.009949	0.011421	0.012944	0.014513	0.016097	0.017746	0.019117	0.019393	0.020066	0.021145	0.022577	0.02377	0.024856	0.45
	8.087291	0.002895	0.002278	0.003113	0.004627	0.006149	0.006855	0.008173	0.009364	0.010535	0.011644	0.013131	0.01426	0.015574	0.016664	0.017448	0.017646	0.018611	0.019737	0.021115	0.022165	0.023405	0.5
		Relative X Position																					
		DATA																					
		Current Change in gear ratio: 0.00305 *All Dimensions in Inches																					
		Relative X: 0																					
		Relative Y: 0																					
		Optimal Change in gear ratio: 0.001803																					
		Relative X: 0																					
		Relative Y: -0.1																					
		Worst Case Change in Gear ratio: 0.025429																					

Figure B.13 14 Tooth 25% Sag Starting Position

		X Position																				Relative Y Position	
		0																					
Y Position		-1.57207	-1.52207	-1.47207	-1.42207	-1.37207	-1.32207	-1.27207	-1.22207	-1.17207	-1.12207	-1.07207	-1.02207	-0.97207	-0.92207	-0.87207	-0.82207	-0.77207	-0.72207	-0.67207	-0.62207	-0.57207	
7.031469		0.004734	0.003548	0.003026	0.002555	0.003261	0.004598	0.005987	0.007424	0.008871	0.010251	0.011565	0.012482	0.013025	0.014497	0.015749	0.017353	0.018696	0.02014	0.021414	0.022807	0.023363	-0.5
7.081469		0.004144	0.003524	0.002864	0.002391	0.003068	0.00462	0.006093	0.007514	0.008919	0.010309	0.011711	0.012242	0.012661	0.013819	0.014857	0.016174	0.017419	0.018718	0.020184	0.02138	0.021608	-0.45
7.131469		0.00408	0.003581	0.003597	0.003526	0.004455	0.00584	0.007373	0.008783	0.010198	0.011666	0.01292	0.013205	0.013898	0.015181	0.016319	0.017448	0.018545	0.019614	0.020733	0.021305	0.021074	-0.4
7.181469		0.004154	0.004775	0.004875	0.004867	0.005847	0.007318	0.008826	0.010128	0.011685	0.013177	0.013975	0.014203	0.015243	0.016524	0.01761	0.018697	0.019964	0.021016	0.02205	0.022472	0.022183	-0.35
7.231469		0.005318	0.005773	0.005986	0.006202	0.007323	0.008848	0.010277	0.011707	0.013155	0.014611	0.014997	0.015599	0.016654	0.017767	0.018984	0.020146	0.021213	0.022272	0.023304	0.023409	0.023168	-0.3
7.281469		0.006193	0.006571	0.006932	0.007452	0.008952	0.01044	0.011828	0.013355	0.01477	0.016062	0.016258	0.016878	0.017964	0.019165	0.020427	0.021508	0.022753	0.023891	0.024413	0.024501	0.024384	-0.25
7.331469		0.00682	0.007214	0.007673	0.009112	0.01062	0.012112	0.013551	0.015006	0.016456	0.017341	0.01755	0.018442	0.019555	0.020711	0.021691	0.022843	0.024003	0.02538	0.0256	0.025716	0.025415	-0.2
7.381469		0.007209	0.008051	0.009477	0.010877	0.012378	0.013857	0.015269	0.016805	0.018221	0.01867	0.018884	0.020046	0.021044	0.022256	0.023422	0.024591	0.025669	0.026711	0.026792	0.026566	0.026477	-0.15
7.431469		0.007788	0.009363	0.010921	0.012489	0.013915	0.015473	0.016987	0.018308	0.019534	0.020013	0.020473	0.021564	0.022746	0.023804	0.024966	0.026258	0.027258	0.028294	0.028032	0.027884	0.027686	-0.1
7.481469		0.007423	0.008742	0.009879	0.011443	0.013015	0.014471	0.016013	0.017355	0.018051	0.018237	0.019047	0.020403	0.021627	0.022881	0.024099	0.025039	0.026118	0.026625	0.026529	0.026343	0.026583	-0.05
7.531469		0.008628	0.009898	0.011144	0.012214	0.013279	0.014554	0.015618	0.016665	0.016737	0.017057	0.018045	0.019119	0.020335	0.02149	0.022584	0.023873	0.024915	0.024842	0.024698	0.024751	0.024954	0
7.581469		0.01001	0.011156	0.012348	0.013592	0.014663	0.015919	0.016814	0.017638	0.017766	0.017719	0.018521	0.019478	0.020402	0.021344	0.022001	0.022925	0.023348	0.02331	0.023262	0.022991	0.023725	0.05
7.631469		0.01144	0.012579	0.013784	0.014933	0.01597	0.017229	0.018325	0.018757	0.018746	0.018909	0.019866	0.020667	0.021532	0.022514	0.023446	0.024247	0.024299	0.02383	0.023557	0.02351	0.023941	0.1
7.681469		0.012838	0.014104	0.015259	0.016258	0.01745	0.018517	0.019819	0.019781	0.020395	0.021212	0.022117	0.022857	0.023836	0.024962	0.025506	0.025384	0.024947	0.024796	0.024904	0.025618	0.15	
7.731469		0.01443	0.015649	0.01665	0.017967	0.019058	0.020148	0.020759	0.020911	0.020744	0.021869	0.022682	0.023555	0.024504	0.025301	0.026243	0.026476	0.026341	0.026068	0.025815	0.02638	0.026846	0.2
7.781469		0.016077	0.01724	0.018405	0.01955	0.02058	0.021917	0.022055	0.021939	0.022225	0.02316	0.024056	0.024805	0.025785	0.026938	0.027847	0.027705	0.027581	0.027167	0.027119	0.027682	0.028339	0.25
7.831469		0.017769	0.018976	0.020279	0.021294	0.022461	0.023286	0.023461	0.023322	0.02379	0.024696	0.02567	0.026587	0.027449	0.028148	0.02898	0.028801	0.028733	0.028302	0.028435	0.02924	0.029855	0.3
7.881469		0.018195	0.019546	0.020898	0.022216	0.023284	0.023872	0.024119	0.024366	0.025296	0.02623	0.027161	0.028055	0.028974	0.030009	0.030278	0.029981	0.029738	0.029303	0.029606	0.030258	0.030391	0.35
7.931469		0.017317	0.01863	0.019835	0.021198	0.022279	0.022459	0.022423	0.022924	0.023978	0.024856	0.026027	0.026885	0.027808	0.028901	0.028892	0.028609	0.028325	0.028128	0.028509	0.02916	0.029721	0.4
7.981469		0.017021	0.017939	0.018939	0.020217	0.020904	0.020956	0.020986	0.021502	0.022628	0.023825	0.024693	0.025705	0.026784	0.027362	0.027275	0.026967	0.026803	0.026508	0.027262	0.027837	0.028538	0.45
8.031469		0.018354	0.019309	0.020215	0.020838	0.020978	0.020826	0.020534	0.021048	0.021627	0.022474	0.023443	0.024606	0.02541	0.025623	0.025514	0.025333	0.024997	0.025334	0.025873	0.026458	0.027303	0.5
		-0.5	-0.45	-0.4	-0.35	-0.3	-0.25	-0.2	-0.15	-0.1	-0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	
		Relative X Position																					
DATA																							
Current Change in gear ratio:		0.018045																			*All Dimensions in Inches		
Relative X:		0																					
Relative Y:		0																					
Optimal Change in gear ratio:		0.002391																					
Relative X:		-0.35																					
Relative Y:		-0.45																					
Worst Case Change in Gear ratio:		0.481372																					

Figure B.14 15 Tooth 25% Sag Starting Position

	X Position																					
															0							
Y Position	-1.62795	-1.57795	-1.52795	-1.47795	-1.42795	-1.37795	-1.32795	-1.27795	-1.22795	-1.17795	-1.12795	-1.07795	-1.02795	-0.97795	-0.92795	-0.87795	-0.82795	-0.77795	-0.72795	-0.67795	-0.62795	Relative Y Position
6.975591	0.012782	0.013779	0.013687	0.013937	0.13483	0.015814	0.017054	0.018475	0.019664	0.020885	0.021491	0.021464	0.021208	0.021363	0.022331	0.022974	0.023952	0.024756	0.025195	0.025149	0.024778	-0.5
7.025591	0.014285	0.014878	0.014819	0.014986	0.015932	0.016732	0.017528	0.018701	0.019437	0.020224	0.020133	0.019892	0.019997	0.019989	0.020919	0.021842	0.022637	0.023631	0.024221	0.023735	0.023348	-0.45
7.075591	0.015752	0.016059	0.015916	0.016408	0.017197	0.018201	0.019001	0.019799	0.020701	0.021556	0.020956	0.020545	0.020336	0.020317	0.021116	0.021525	0.022039	0.022607	0.022276	0.022254	0.021865	-0.4
7.125591	0.017133	0.017148	0.017061	0.017846	0.018721	0.019495	0.0203	0.021179	0.022029	0.022505	0.022207	0.021776	0.02128	0.021668	0.022429	0.022871	0.023394	0.023943	0.023824	0.023171	0.022366	-0.35
7.175591	0.018373	0.018289	0.018473	0.019296	0.020225	0.020981	0.021938	0.022728	0.023596	0.023763	0.023321	0.022843	0.022657	0.023205	0.023714	0.024472	0.024928	0.025393	0.02493	0.024285	0.02356	-0.3
7.225591	0.019507	0.019532	0.020257	0.021049	0.021766	0.022748	0.02348	0.024239	0.025068	0.024836	0.024483	0.02414	0.024012	0.024767	0.025191	0.02587	0.026318	0.026413	0.025937	0.025331	0.024617	-0.25
7.275591	0.02097	0.020991	0.021778	0.022516	0.023509	0.024393	0.025133	0.02594	0.026339	0.02603	0.025501	0.025357	0.025459	0.026134	0.026879	0.027321	0.027909	0.027783	0.027305	0.026803	0.026008	-0.2
7.325591	0.020374	0.020575	0.021533	0.022517	0.023671	0.024444	0.025613	0.026624	0.026664	0.026459	0.026178	0.025751	0.026093	0.02685	0.02743	0.02791	0.028398	0.027747	0.027066	0.026275	0.025391	-0.15
7.375591	0.018695	0.019431	0.02048	0.021537	0.02245	0.023614	0.02447	0.025347	0.02508	0.024897	0.024565	0.024424	0.025098	0.025464	0.026382	0.02672	0.026984	0.026385	0.02577	0.025023	0.024133	-0.1
7.425591	0.017253	0.018101	0.01942	0.020412	0.02147	0.022518	0.023579	0.023639	0.023464	0.023407	0.023124	0.023233	0.023932	0.024578	0.025082	0.025657	0.025547	0.024957	0.024213	0.023473	0.022936	-0.05
7.475591	0.018028	0.01856	0.01929	0.019916	0.020664	0.021448	0.022395	0.022275	0.021954	0.021801	0.021489	0.022096	0.022769	0.023429	0.024071	0.024551	0.024035	0.023376	0.022707	0.02212	0.021479	0
7.525591	0.019429	0.019926	0.020419	0.021193	0.021907	0.022444	0.02268	0.022163	0.021473	0.020988	0.020411	0.020669	0.021474	0.022445	0.023074	0.023093	0.022622	0.022032	0.021415	0.020602	0.019967	0.05
7.575591	0.020705	0.02138	0.021979	0.022615	0.023344	0.023839	0.023809	0.023163	0.022643	0.021961	0.021843	0.022071	0.022373	0.022819	0.023255	0.022742	0.021768	0.020857	0.019966	0.019322	0.018913	0.1
7.625591	0.022303	0.022868	0.023319	0.02408	0.024879	0.025381	0.024894	0.024272	0.023851	0.02308	0.022983	0.023582	0.023888	0.024273	0.024576	0.023866	0.02299	0.022233	0.021418	0.020511	0.019913	0.15
7.675591	0.023976	0.024435	0.025188	0.025769	0.026361	0.026441	0.026051	0.025539	0.025016	0.024376	0.024589	0.025024	0.025441	0.025839	0.025814	0.025	0.024081	0.023402	0.022789	0.021948	0.021818	0.2
7.725591	0.025657	0.02623	0.026776	0.027268	0.027932	0.027622	0.027192	0.026523	0.026163	0.025732	0.025938	0.026522	0.026998	0.027315	0.026811	0.026266	0.025558	0.0248	0.024022	0.023171	0.023411	0.25
7.775591	0.024725	0.025651	0.026351	0.0272	0.027857	0.027458	0.027093	0.026578	0.026052	0.026005	0.026551	0.026899	0.027408	0.027548	0.026731	0.025962	0.025019	0.024272	0.023476	0.022564	0.022675	0.3
7.825591	0.023536	0.024517	0.025324	0.025958	0.026371	0.025919	0.025651	0.025184	0.024597	0.024887	0.025401	0.025919	0.026185	0.026105	0.025455	0.024686	0.023823	0.023009	0.022031	0.021663	0.02178	0.35
7.875591	0.022543	0.0233	0.024148	0.02502	0.024539	0.024348	0.024032	0.023773	0.023274	0.023765	0.024225	0.024918	0.02535	0.024716	0.02406	0.023207	0.022361	0.021667	0.020812	0.020752	0.020703	0.4
7.925591	0.021263	0.02236	0.023237	0.02348	0.023325	0.022965	0.022505	0.022324	0.022256	0.022676	0.023386	0.023747	0.023704	0.02317	0.0224	0.021774	0.02103	0.02023	0.019484	0.019677	0.019973	0.45
7.975591	0.022262	0.022906	0.023249	0.02267	0.02206	0.021301	0.021124	0.020643	0.020902	0.021613	0.022038	0.022643	0.022387	0.021868	0.021173	0.020333	0.01956	0.018948	0.018663	0.018721	0.018924	0.5
	-0.5	-0.45	-0.4	-0.35	-0.3	-0.25	-0.2	-0.15	-0.1	-0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	
	Relative X Position																					
DATA																						
Current Change in gear ratio:		0.021489		*All Dimensions in Inches																		
Relative X:		0																				
Relative Y:		0																				
Optimal Change in gear ratio:		0.012782																				
Relative X:		-0.5																				
Relative Y:		-0.5																				
Worst Case Change in Gear ratio:		0.13483																				

Figure B.15 16 Tooth 25% Sag Starting Position

	X Position																								
																					0				
Y Position	-1.68388	-1.63388	-1.58388	-1.53388	-1.48388	-1.43388	-1.38388	-1.33388	-1.28388	-1.23388	-1.18388	-1.13388	-1.08388	-1.03388	-0.98388	-0.93388	-0.88388	-0.83388	-0.78388	-0.73388	-0.68388	Relative Y Position			
6.919665	0.019018	0.018478	0.018215	0.018006	0.01805	0.018904	0.019796	0.020367	0.021194	0.021435	0.020963	0.020435	0.019981	0.019327	0.018749	0.018122	0.019283	0.019868	0.019112	0.018116	0.017274	-0.5			
6.969665	0.020225	0.019402	0.019029	0.018211	0.017931	0.018417	0.018815	0.019475	0.020173	0.019957	0.019531	0.018903	0.018599	0.017826	0.017583	0.018112	0.018459	0.018614	0.017818	0.016765	0.015805	-0.45			
7.019665	0.021244	0.020713	0.019982	0.019414	0.019454	0.019713	0.019979	0.020344	0.020703	0.019911	0.019065	0.018215	0.017176	0.016726	0.016389	0.017051	0.017168	0.017232	0.01641	0.01554	0.014649	-0.4			
7.069665	0.022356	0.021692	0.021226	0.020682	0.020799	0.021202	0.021542	0.021873	0.022207	0.021166	0.020067	0.019328	0.018516	0.01764	0.017348	0.017469	0.017629	0.016935	0.015738	0.01457	0.013455	-0.35			
7.119665	0.023644	0.022827	0.022578	0.022096	0.022447	0.022713	0.022921	0.02315	0.023409	0.022205	0.021399	0.020565	0.019712	0.018825	0.018812	0.019069	0.019156	0.018044	0.017098	0.01608	0.014841	-0.3			
7.169665	0.025075	0.024146	0.023987	0.023397	0.023844	0.024077	0.024285	0.024519	0.024788	0.023584	0.022781	0.021949	0.021098	0.020228	0.020241	0.020495	0.020694	0.020379	0.019407	0.018422	0.017399	0.016163	-0.25		
7.219665	0.026594	0.025595	0.025505	0.024805	0.025254	0.025497	0.025681	0.025884	0.026107	0.024903	0.024101	0.02327	0.02242	0.02155	0.021568	0.021822	0.021994	0.021694	0.020694	0.019694	0.018585	0.017385	0.016186	0.014829	-0.2
7.269665	0.028243	0.027133	0.027056	0.026252	0.026703	0.026946	0.027091	0.027246	0.027411	0.026207	0.025405	0.024576	0.023728	0.02286	0.022873	0.023127	0.023299	0.023004	0.021994	0.020994	0.019885	0.018685	0.017485	0.016186	-0.15
7.319665	0.029947	0.028738	0.028672	0.027768	0.028219	0.028462	0.028597	0.028742	0.028897	0.027693	0.026891	0.026062	0.025214	0.024356	0.024369	0.024623	0.024795	0.024504	0.023494	0.022494	0.021294	0.020094	0.018894	0.017494	-0.1
7.369665	0.031696	0.030388	0.030332	0.029328	0.029779	0.029992	0.030097	0.030192	0.030287	0.029083	0.028281	0.027452	0.026604	0.025746	0.025759	0.026013	0.026185	0.025894	0.024884	0.023884	0.022684	0.021484	0.020284	0.018884	-0.05
7.419665	0.03349	0.032083	0.032037	0.030933	0.031384	0.031597	0.031692	0.031777	0.031862	0.030658	0.029856	0.029027	0.028179	0.027321	0.027334	0.027588	0.02776	0.027469	0.026459	0.025459	0.024259	0.023059	0.021859	0.020459	0
7.469665	0.03529	0.033783	0.033747	0.032543	0.032994	0.033207	0.033292	0.033377	0.033462	0.032258	0.031456	0.030627	0.029779	0.028921	0.028934	0.029188	0.02936	0.029069	0.028059	0.027059	0.025859	0.024659	0.023459	0.022059	0.05
7.519665	0.03709	0.035483	0.035457	0.034153	0.034604	0.034817	0.034892	0.034967	0.035042	0.033838	0.033036	0.032207	0.031359	0.030501	0.030514	0.030768	0.03094	0.030649	0.029639	0.028639	0.027439	0.026239	0.025039	0.023639	0.1
7.569665	0.03889	0.037183	0.037167	0.035779	0.03623	0.036443	0.036518	0.036593	0.036668	0.035464	0.034662	0.033833	0.032985	0.032127	0.03214	0.032394	0.032566	0.032275	0.031265	0.030265	0.029065	0.027865	0.026665	0.025265	0.15
7.619665	0.04069	0.038883	0.038877	0.037399	0.03785	0.038063	0.038138	0.038213	0.038288	0.037084	0.036282	0.035453	0.034605	0.033747	0.03376	0.034014	0.034186	0.033895	0.032885	0.031885	0.030685	0.029485	0.028285	0.026885	0.2
7.669665	0.04249	0.040583	0.040587	0.038999	0.03945	0.039663	0.039738	0.039813	0.039888	0.038684	0.037882	0.037053	0.036205	0.035347	0.03536	0.035614	0.035786	0.035495	0.034485	0.033485	0.032285	0.031085	0.029885	0.028485	0.25
7.719665	0.04429	0.042283	0.042297	0.040599	0.04105	0.041263	0.041338	0.041413	0.041488	0.040284	0.039482	0.038653	0.037805	0.036947	0.03696	0.037214	0.037386	0.037095	0.036085	0.035085	0.033885	0.032685	0.031485	0.030085	0.3
7.769665	0.04609	0.043983	0.043997	0.042199	0.04265	0.042863	0.042938	0.043013	0.043088	0.041884	0.041082	0.040253	0.039405	0.038547	0.03856	0.038814	0.038986	0.038695	0.037685	0.036685	0.035485	0.034285	0.033085	0.031685	0.35
7.819665	0.04789	0.045683	0.045697	0.043799	0.04425	0.044463	0.044538	0.044613	0.044688	0.043484	0.042682	0.041853	0.041005	0.040147	0.04016	0.040414	0.040586	0.040295	0.039285	0.038285	0.037085	0.035885	0.034685	0.033285	0.4
7.869665	0.04969	0.047383	0.047397	0.045399	0.04585	0.046063	0.046138	0.046213	0.046288	0.045084	0.044282	0.043453	0.042605	0.041747	0.04176	0.042014	0.042186	0.041895	0.040885	0.039885	0.038685	0.037485	0.036285	0.034885	0.45
7.919665	0.05149	0.049083	0.049097	0.046999	0.04745	0.047663	0.047738	0.047813	0.047888	0.046684	0.045882	0.045053	0.044205	0.043347	0.04336	0.043614	0.043786	0.043495	0.042485	0.041485	0.040285	0.039085	0.037885	0.036485	0.5
	-0.5	-0.45	-0.4	-0.35	-0.3	-0.25	-0.2	-0.15	-0.1	-0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5				
	Relative X Position																								
DATA																									
Current Change in gear ratio:		0.016709 *All Dimensions in Inches																							
Relative X:		0																							
Relative Y:		0																							
Optimal Change in gear ratio:		0.003738																							
Relative X:		0.5																							
Relative Y:		0.5																							
Worst Case Change In Gear ratio:		0.025156																							

Figure B.16 17 Tooth 25% Sag Starting Position

		X Position																					
																						0	
Y Position		-1.73984	-1.68984	-1.63984	-1.58984	-1.53984	-1.48984	-1.43984	-1.38984	-1.33984	-1.28984	-1.23984	-1.18984	-1.13984	-1.08984	-1.03984	-0.98984	-0.93984	-0.88984	-0.83984	-0.78984	-0.73984	Relative Y Position
	6.863702	0.017151	0.016476	0.016192	0.015757	0.015088	0.014483	0.014854	0.015221	0.015333	0.014516	0.01364	0.012672	0.011709	0.010761	0.009783	0.00859	0.008487	0.00768	0.006379	0.005034	0.003738	-0.5
	6.913702	0.017552	0.01669	0.01545	0.014508	0.013921	0.013493	0.013858	0.014218	0.014023	0.013181	0.012243	0.01138	0.010457	0.00947	0.008634	0.007608	0.007666	0.00635	0.005187	0.003886	0.003043	-0.45
	6.963702	0.018818	0.017752	0.016745	0.015691	0.01456	0.013996	0.013928	0.013881	0.012815	0.011871	0.010994	0.010149	0.009214	0.008272	0.007502	0.006701	0.006383	0.179576	0.004016	0.003168	0.003584	-0.4
	7.013702	0.020061	0.01913	0.018082	0.017006	0.015984	0.015588	0.015514	0.015213	0.013999	0.012848	0.011549	0.010209	0.009055	0.007743	0.006397	0.005712	0.005289	0.004975	0.004575	0.004111	0.004267	-0.35
	7.063702	0.020556	0.019748	0.018929	0.018071	0.017147	0.017275	0.017291	0.016609	0.015375	0.014114	0.012788	0.011693	0.010342	0.009141	0.007747	0.007175	0.006694	0.006086	0.005358	0.004651	0.004144	-0.3
	7.113702	0.01903	0.018373	0.017398	0.016717	0.01616	0.016262	0.016351	0.015311	0.014182	0.013015	0.011865	0.01051	0.009292	0.007959	0.007449	0.007062	0.006258	0.005457	0.004596	0.003658	0.002952	-0.25
	7.163702	0.017777	0.016874	0.016156	0.015421	0.015034	0.015207	0.015006	0.01393	0.012844	0.011719	0.010553	0.009305	0.008108	0.007469	0.007275	0.006598	0.005591	0.004593	0.003511	0.002386	0.002332	-0.2
	7.213702	0.016314	0.015575	0.014741	0.01393	0.014048	0.01417	0.013619	0.012629	0.011541	0.010432	0.009299	0.008057	0.007175	0.007006	0.006873	0.005781	0.004604	0.003402	0.002284	0.002371	0.003347	-0.15
	7.263702	0.015021	0.014199	0.013403	0.01278	0.013074	0.013201	0.01223	0.011191	0.010196	0.009193	0.008057	0.00685	0.006475	0.00645	0.006008	0.00479	0.003531	0.00254	0.002649	0.00363	0.004841	-0.1
	7.313702	0.013677	0.013005	0.012198	0.011796	0.012105	0.011775	0.011001	0.009882	0.008898	0.007907	0.006882	0.005729	0.005583	0.005601	0.00484	0.003691	0.002502	0.002806	0.004026	0.005261	0.006537	-0.05
	7.363702	0.012332	0.011644	0.010985	0.010864	0.011093	0.010542	0.00971	0.008624	0.007677	0.006684	0.005662	0.004643	0.004418	0.004473	0.003457	0.002365	0.00301	0.004228	0.005449	0.006732	0.008015	0
	7.413702	0.012739	0.011538	0.010295	0.009932	0.01021	0.009361	0.008311	0.007422	0.00646	0.005506	0.004632	0.003479	0.002975	0.002759	0.00209	0.003062	0.004282	0.0055	0.006786	0.008047	0.009329	0.05
	7.463702	0.013913	0.012751	0.011805	0.011609	0.010948	0.009747	0.008537	0.007373	0.006287	0.005074	0.004022	0.002938	0.00171	0.001799	0.002996	0.004178	0.005428	0.006661	0.007912	0.00919	0.010375	0.1
	7.513702	0.014056	0.013039	0.012574	0.012526	0.011513	0.010284	0.009022	0.007811	0.006623	0.005422	0.004241	0.003035	0.002293	0.002309	0.003457	0.004684	0.005787	0.006981	0.008044	0.009012	0.009884	0.15
	7.563702	0.01275	0.011761	0.011655	0.011397	0.010251	0.009049	0.00783	0.006581	0.005327	0.004071	0.00285	0.0026	0.002427	0.002245	0.002879	0.003916	0.005042	0.00598	0.006865	0.007795	0.008843	0.2
	7.613702	0.011515	0.01068	0.010736	0.009997	0.008849	0.007703	0.006551	0.00536	0.004147	0.003458	0.003144	0.002906	0.002972	0.002824	0.003228	0.004387	0.005486	0.006586	0.007682	0.008776	0.009813	0.25
	7.663702	0.010296	0.009706	0.009787	0.008709	0.007548	0.00643	0.005274	0.004067	0.003887	0.003657	0.003484	0.004025	0.004031	0.003983	0.004282	0.005446	0.006554	0.007605	0.00873	0.00996	0.010523	0.3
	7.713702	0.008942	0.008789	0.008466	0.00745	0.006332	0.005228	0.004111	0.003934	0.003883	0.003779	0.004594	0.004909	0.004911	0.004905	0.005366	0.006515	0.007668	0.008793	0.009944	0.011108	0.011233	0.35
0	7.763702	0.007741	0.007889	0.007158	0.006192	0.005033	0.004006	0.00371	0.00364	0.004109	0.00483	0.005469	0.00555	0.005603	0.005633	0.006497	0.00766	0.008825	0.009989	0.011231	0.012145	0.012067	0.4
	7.813702	0.006887	0.006877	0.005879	0.004855	0.004011	0.002894	0.003499	0.004232	0.004964	0.005499	0.005868	0.005986	0.00607	0.006557	0.007742	0.008937	0.010143	0.011302	0.012548	0.01301	0.013047	0.45
	7.863702	0.006127	0.005569	0.004682	0.004101	0.00377	0.003527	0.004237	0.004853	0.005385	0.00577	0.006017	0.006188	0.006894	0.008039	0.009172	0.010315	0.0115	0.012753	0.013973	0.013965	0.014088	0.5
		-0.5	-0.45	-0.4	-0.35	-0.3	-0.25	-0.2	-0.15	-0.1	-0.05	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	
		Relative X Position																					
DATA																							
Current Change in gear ratio:		0.005662																				*All Dimensions in Inches	
Relative X:		0																					
Relative Y:		0																					
Optimal Change in gear ratio:		0.00171																					
Relative X:		0.1																					
Relative Y:		0.1																					
Worst Case Change in Gear ratio:		0.179576																					

Figure B.17 18 Tooth 25% Sag Starting Position

B.3 – Idler Tooth Count Sweep Results

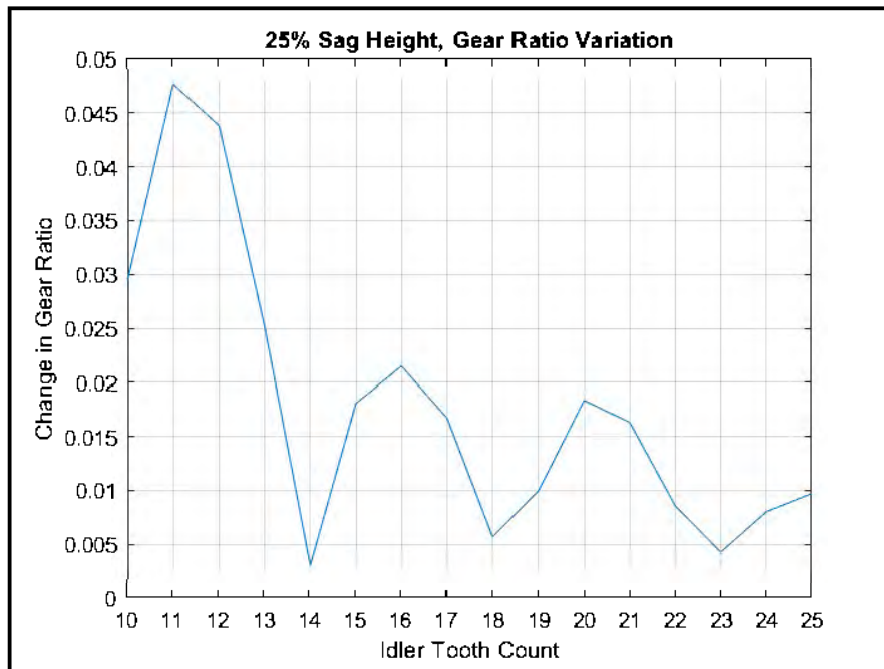


Figure B.18 25% Sag Tooth Count Range

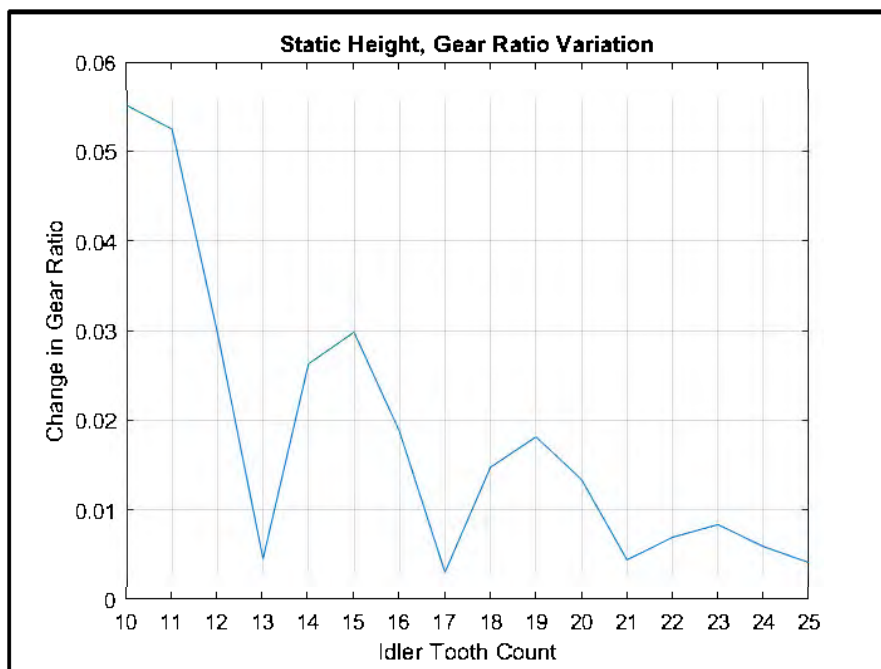
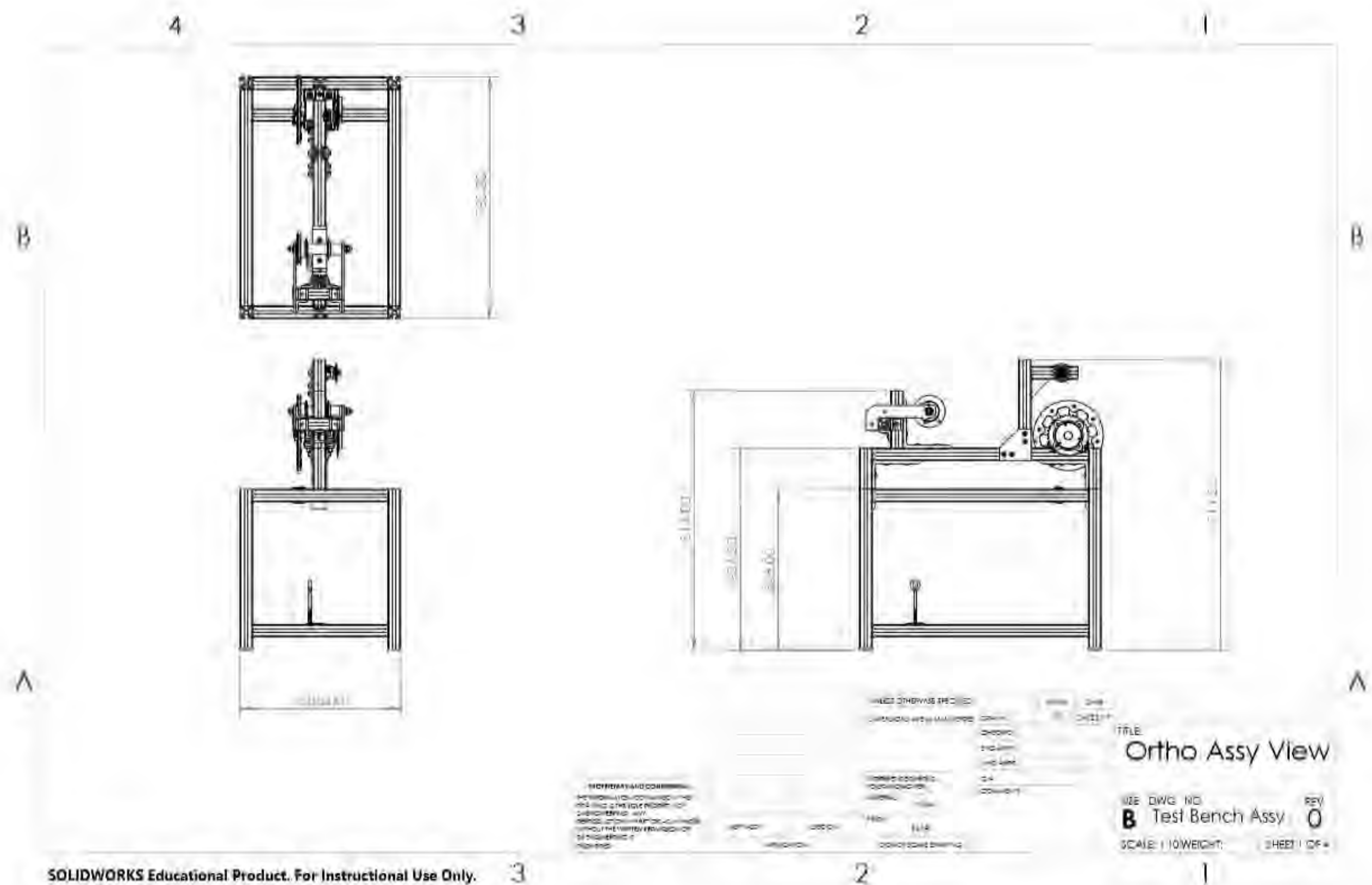
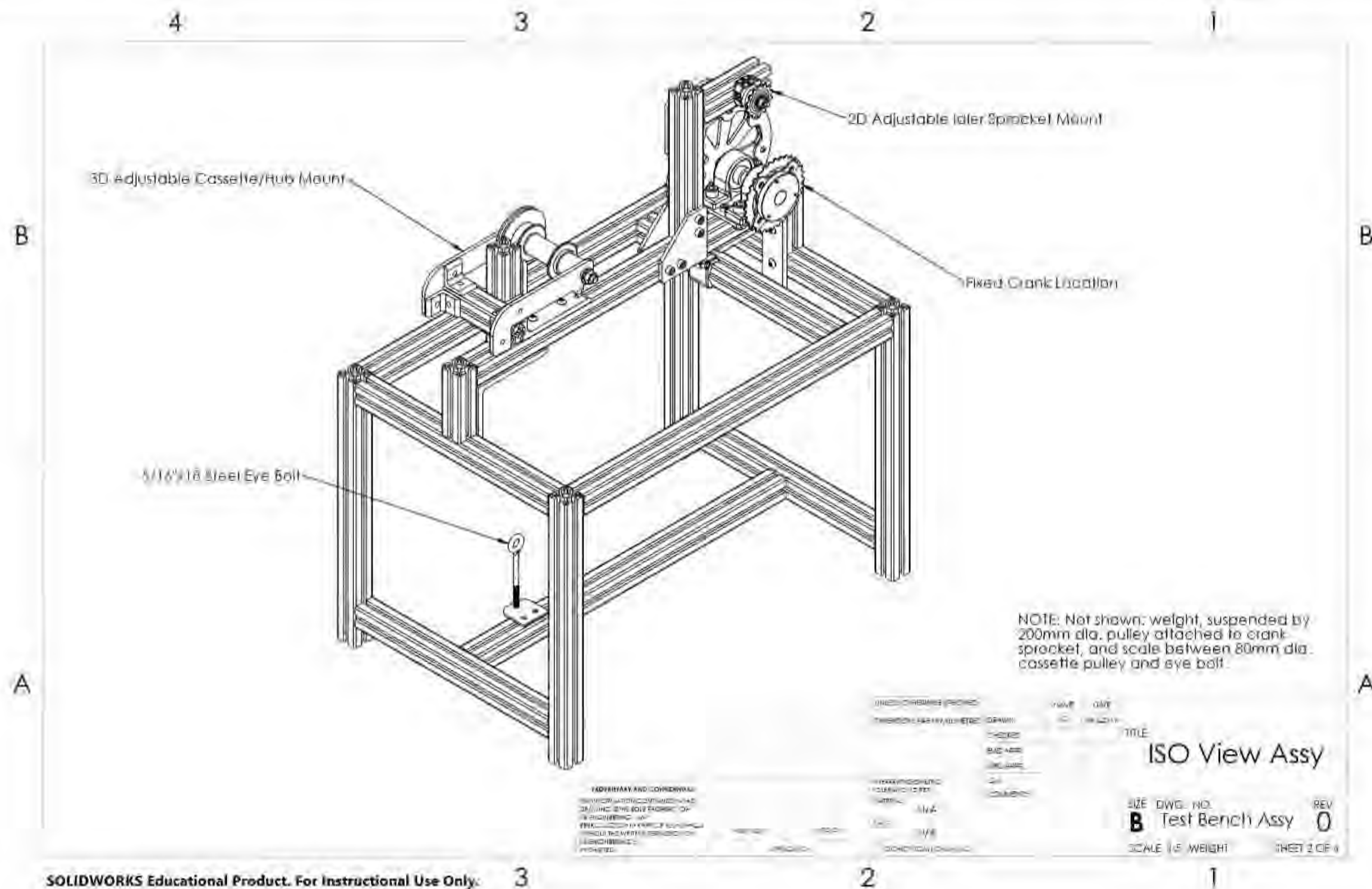
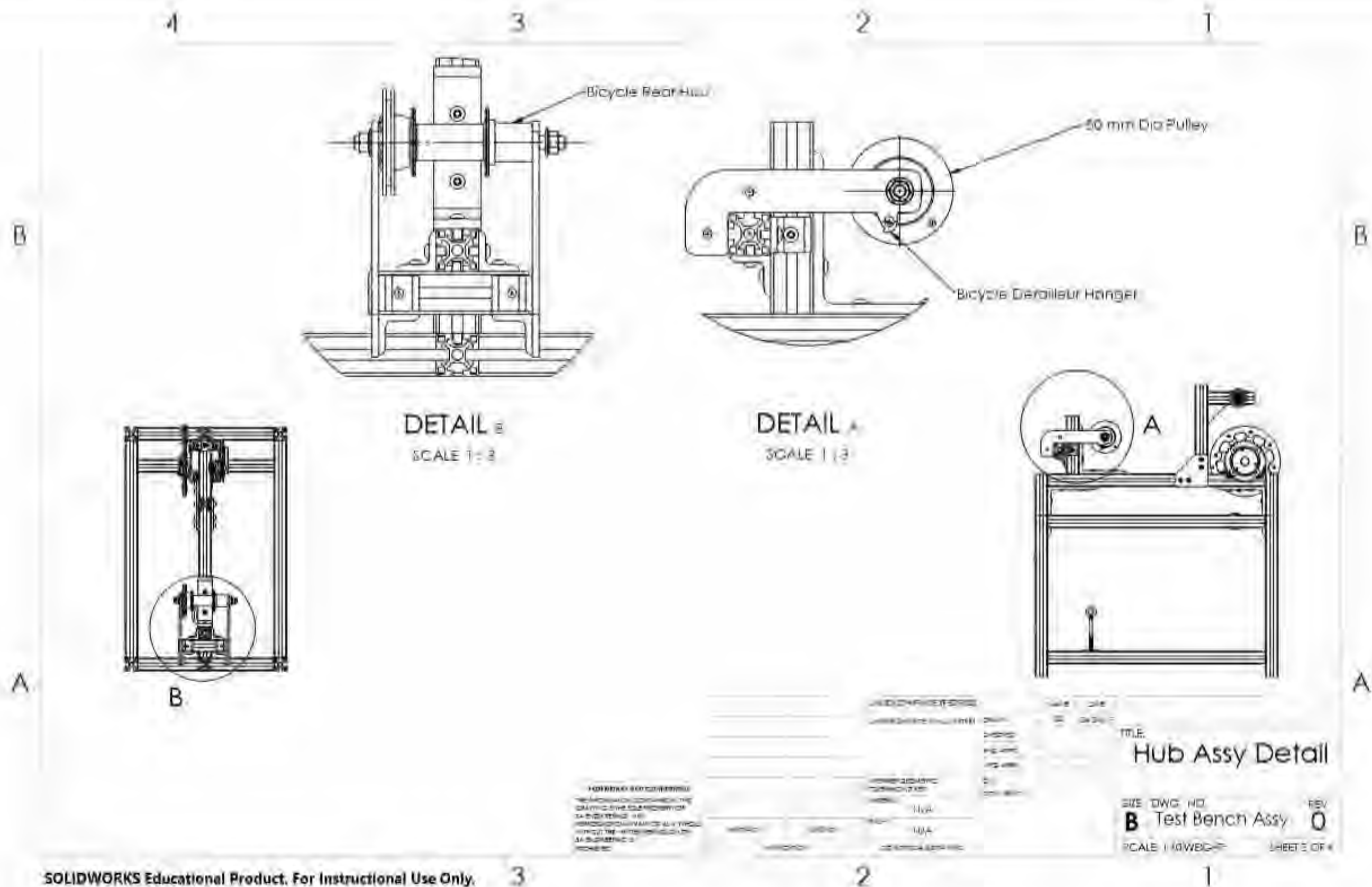


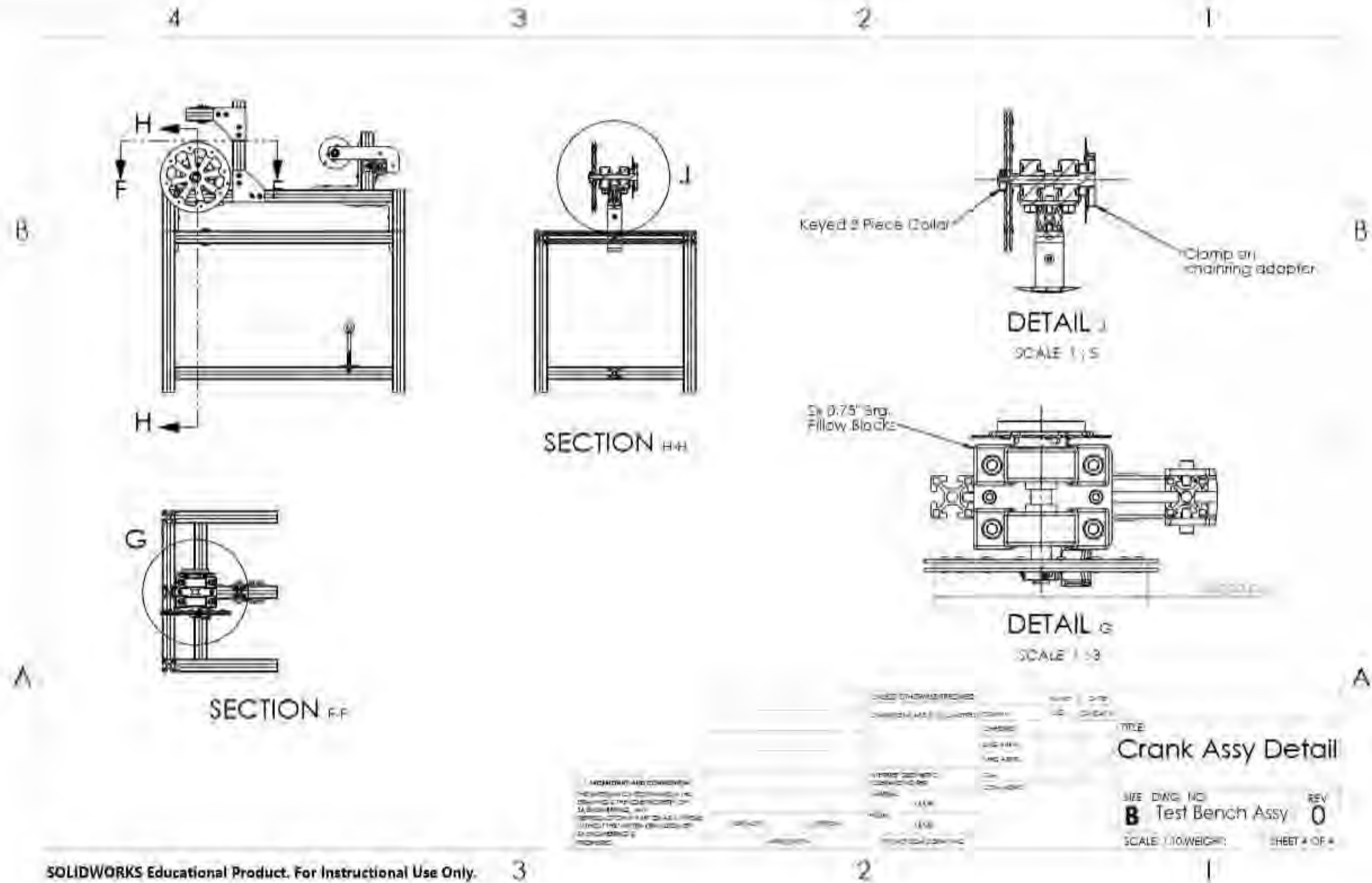
Figure B.19 Static Height Tooth Count Range

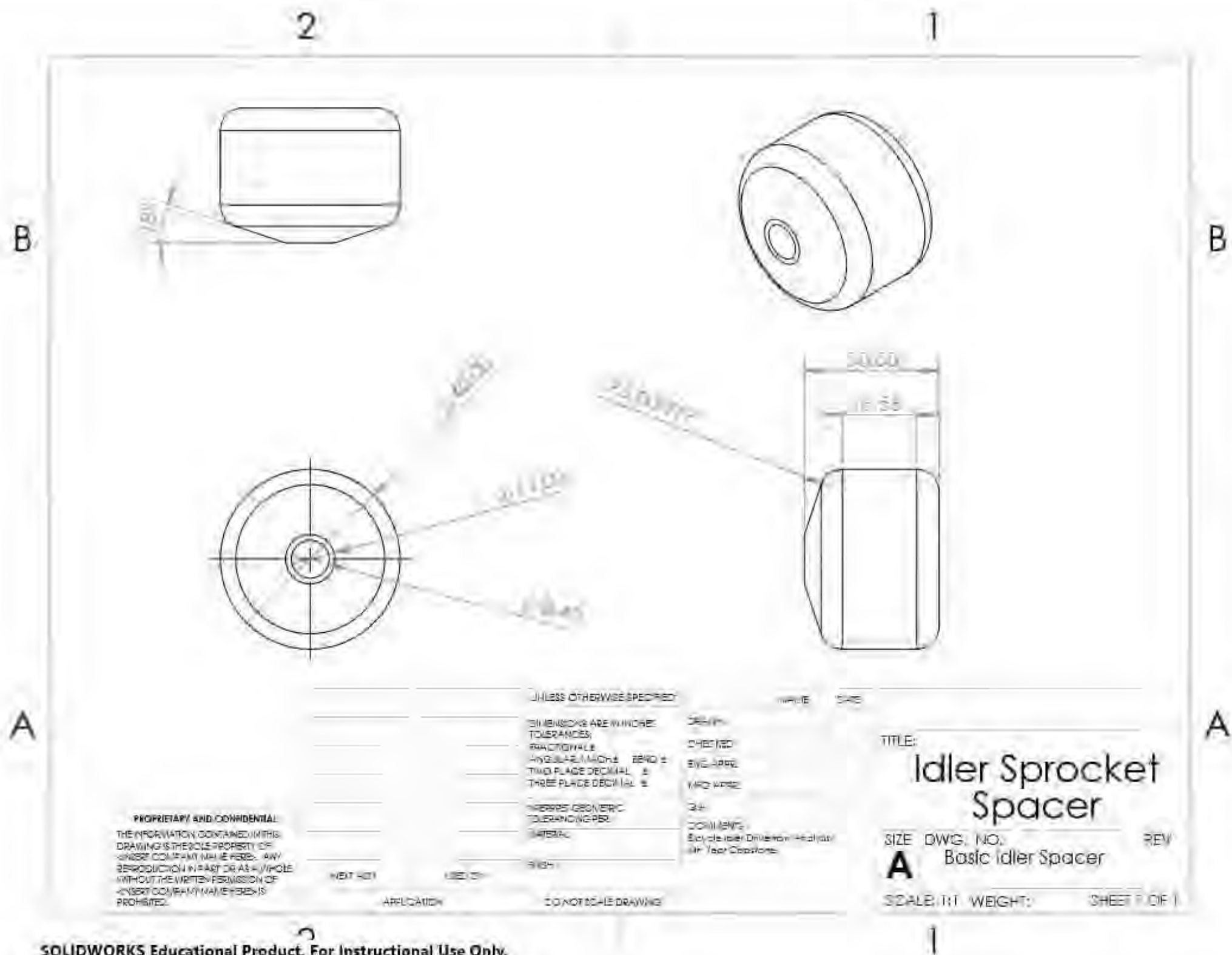
Appendix C – Test Bench Drawings

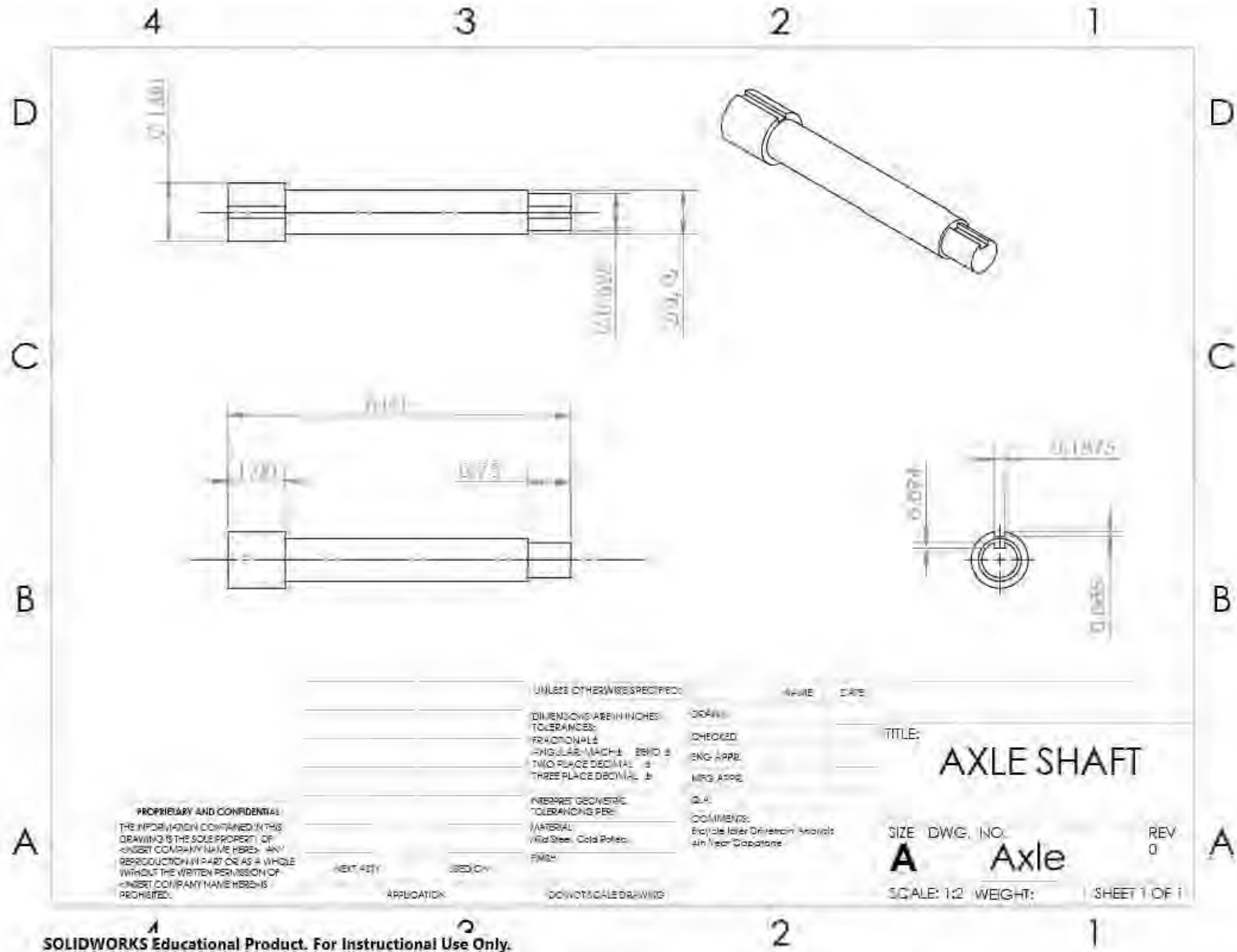






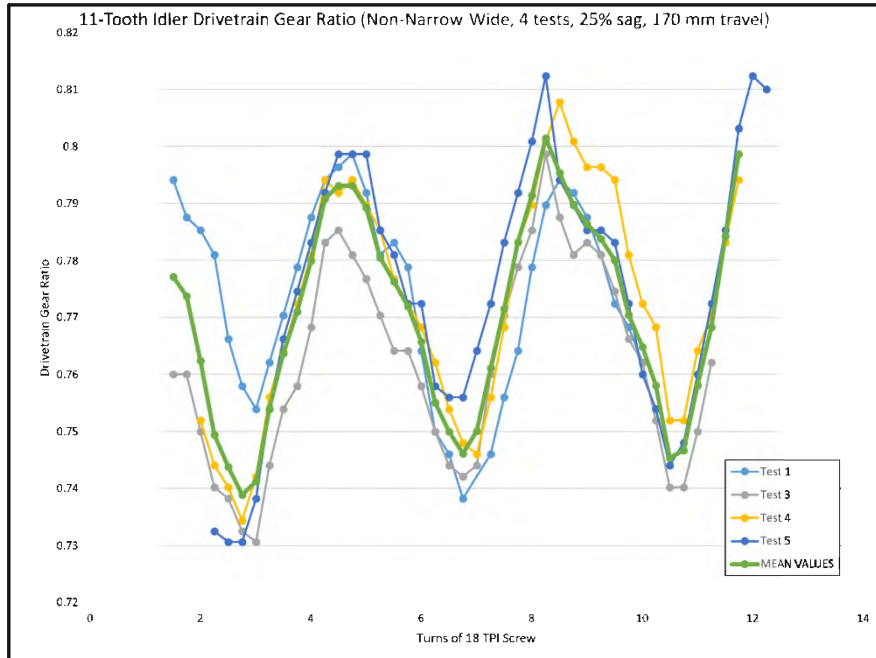




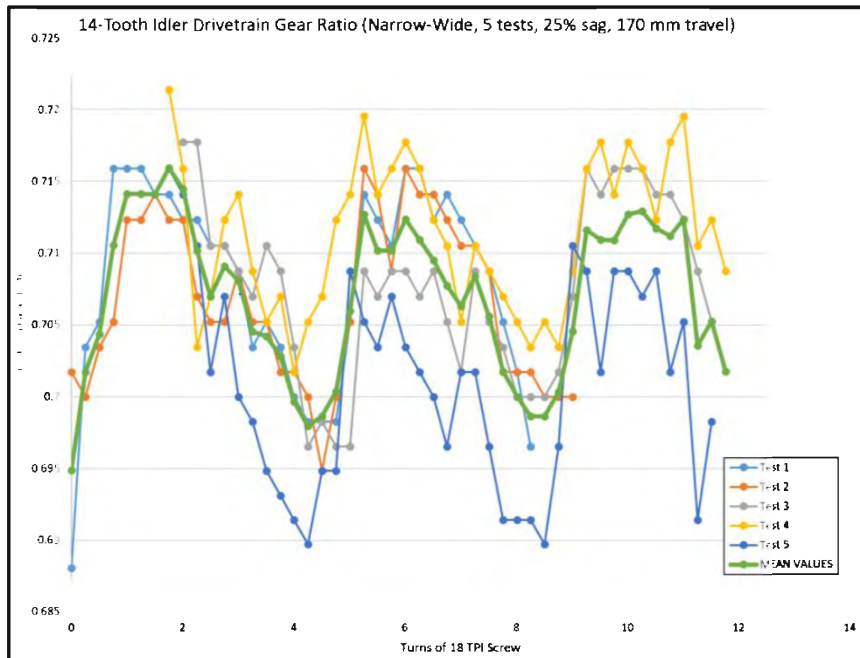


Appendix D – Test Bench Data

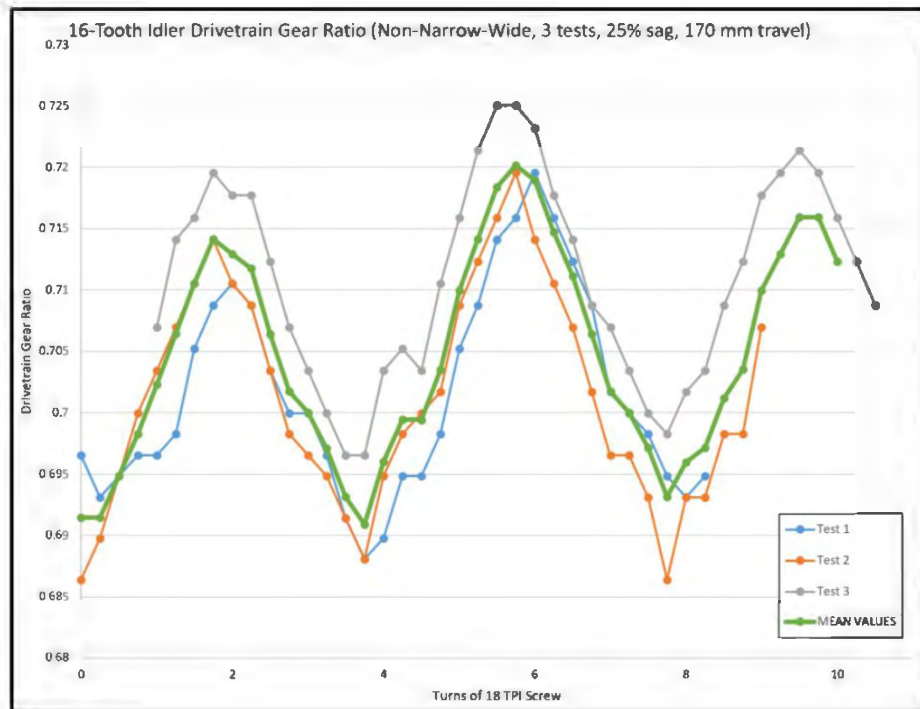
D.1 – 11-Tooth Idler - Plastic, Long-Tooth, Non-Narrow-Wide 25% Sag



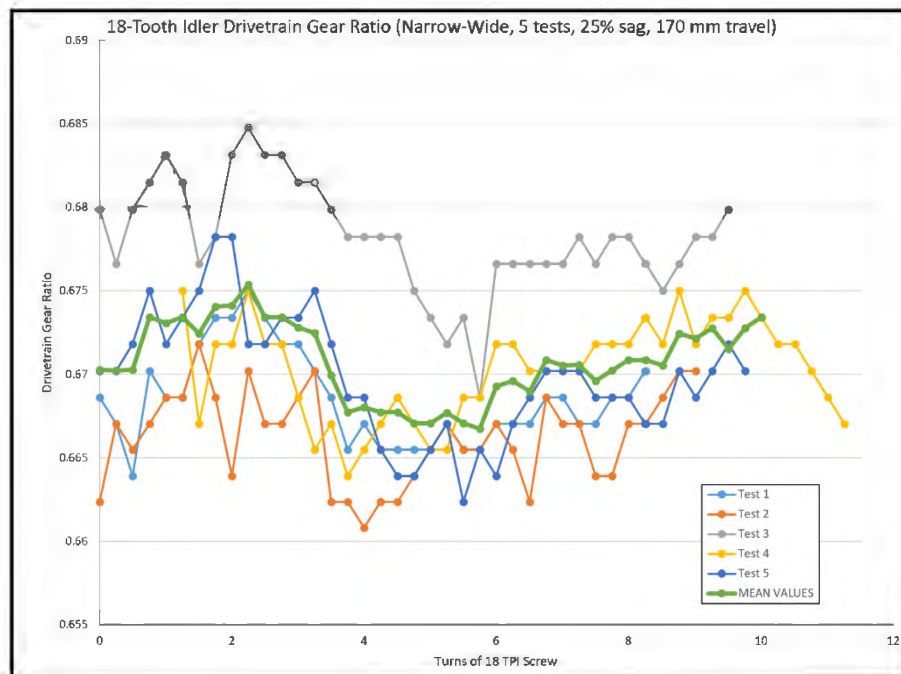
D.2 – 14-Tooth Idler - Plastic, Narrow-Wide, 25% Sag, 25 lb



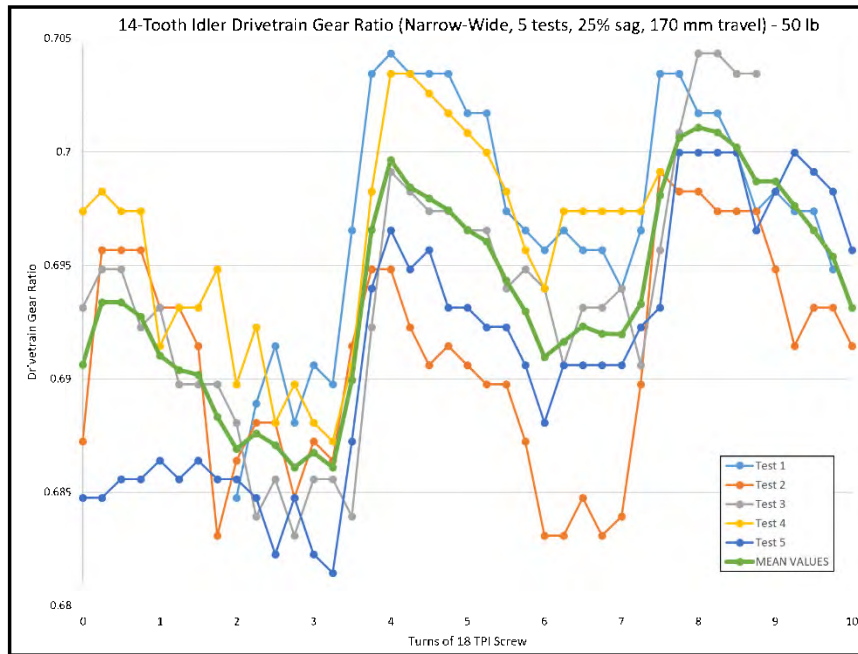
D.3 – 16-Tooth Idler - Aluminum, Non-Narrow-Wide, 25% Sag



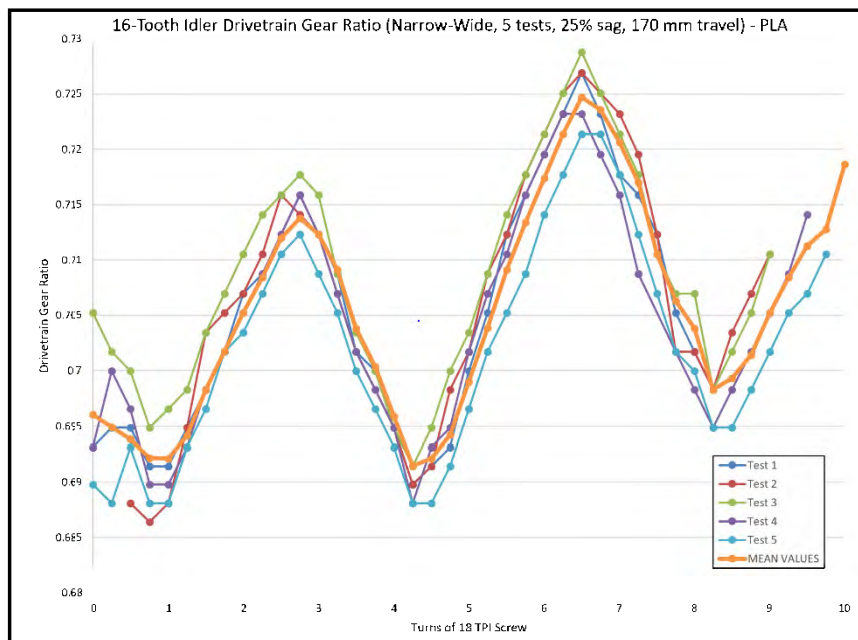
D.4 – 18-Tooth Idler - Plastic, Narrow-Wide, 25% Sag, 25 lb



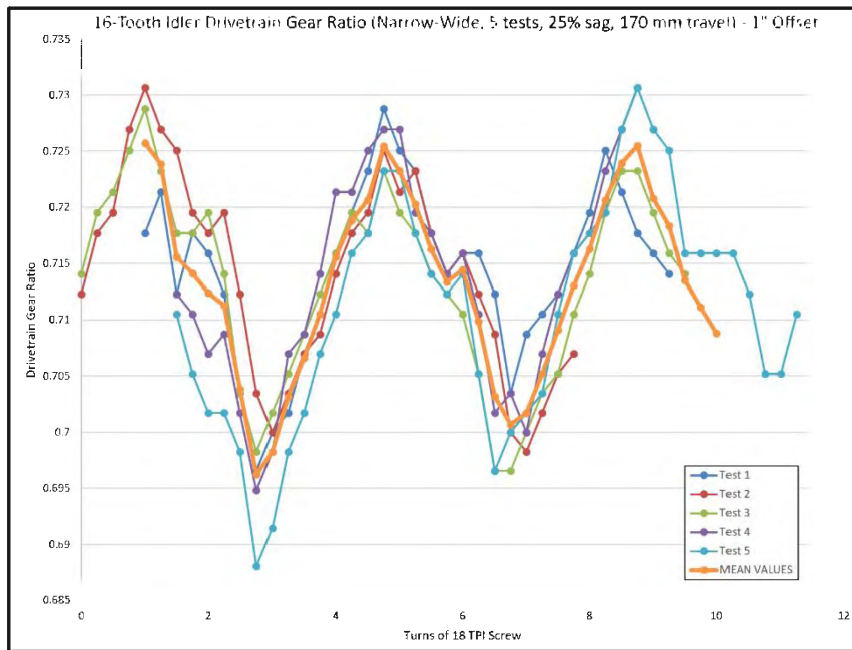
D.5 – 14-Tooth Idler - Plastic, Narrow-Wide, 25% Sag, 50 lb



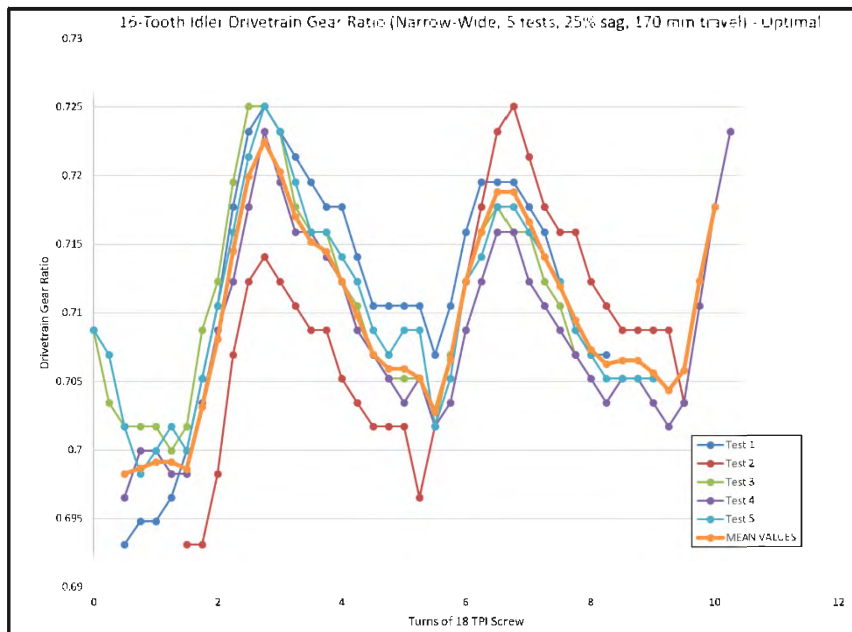
D.6 – 16-Tooth Idler - Aluminum, Narrow-Wide, 25% Sag, 25 lb.



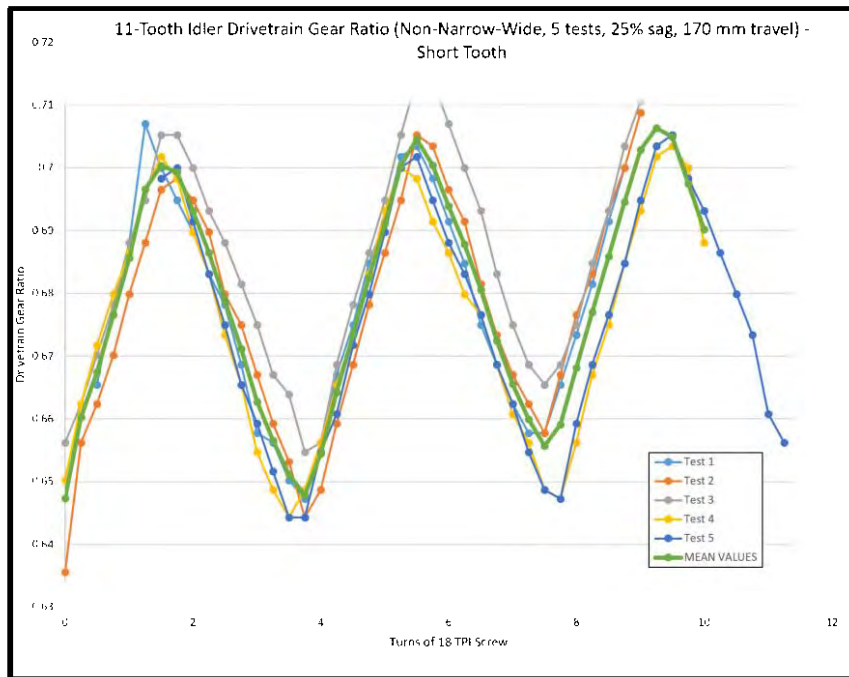
D.7 – 16-Tooth Idler - Plastic, Narrow-Wide, 25% Sag, 25 lb, 1" Chain Line Offset



D.8 – 16-Tooth Idler - Plastic, Non-Narrow-Wide, 25% Sag, 25 lb, Optimal Position



D.9 – 11-Tooth Idler - Plastic, Non-Narrow-Wide, 25% Sag, 25 lb, Short Tooth



Appendix E – Request for Proposal

Project Name: Bicycle Idler Sprocket Drivetrain Analysis

Company Name: Jelly Bean Consulting

Address: 3700 Willingdon Avenue

Burnaby BC, Canada

Procurement Contact Person (PCP): Jelly B Kames

Telephone number of PCP: (604) 737-7135

Email address of PCP: jkames@jellybeanconsulting.ca

Jelly Bean Consulting is a small engineering consulting firm operating out of Burnaby, BC. The firm was founded in 2014, and specializes in high performance sports equipment. Our company prides itself on our high quality of work, and excellent customer service.

The main goal for the project is to study and quantify losses and drivetrain vibrations associated with the polygonal shape of the modern bicycle idler pulley in a high pivot application. The deliverables are to be a Matlab analysis, a Solidworks motion study, a bench prototype, a rolling test bike, as well as any test results.

Below are details on the various deadlines for proposal submission:

- The request for proposal will be sent out on October 16th, 2018.
 - Questions should be submitted prior to October 20th, 2018 to ensure responses before the deadline.
 - Questions will be responded to prior to October 22, 2018.
 - The proposals should be submitted to the provided email address no later than Tuesday October 23, 2018.
 - The desired applicant will be selected by Jelly Bean Consulting on Friday October 26, 2018.
-

The project must be completed prior to May 2019's MECH Project Expo. This allows 7 months for project completion.

The following components must be included in any project proposal in order for it to be considered:

- Background information, and team member information.
- Project objective.
- Review of background and supporting information pertaining to the project.
- Project specifics, describing the method used to complete the project.
- Justification and motivation of the team.
- The deliverables of the project.
- A proposed milestone schedule.
- Technical requirements to be met.
- Limits and exclusions.
- Project Work Breakdown Structure (WBS).
- Responsibility Assignment Matrix.
- Project schedule.
- Project budget.
- End-of-Life plan.

The project team will be evaluated on the following key points:

- Previous industry experience
 - Competitive budgeting
 - Time flexibility, and ability to commit to the project
 - Proposed timeline for project
-

Some potential roadblocks that should be addressed include:

- Prototype and test bed creation may be delayed due to material and machine time availability.
- Availability of all involved parties is subject to change.
- Scope of project may be subject to change to encompass unforeseen problems or challenges.

The project has a budget of no more than \$1000.00 CAD. This is to facilitate any prototype creation, data logging equipment, and rolling test bikes necessary to gather data and quantify results.

Kelly James

Jordan Donaldson

Denton Anderson

Appendix F – Management Items

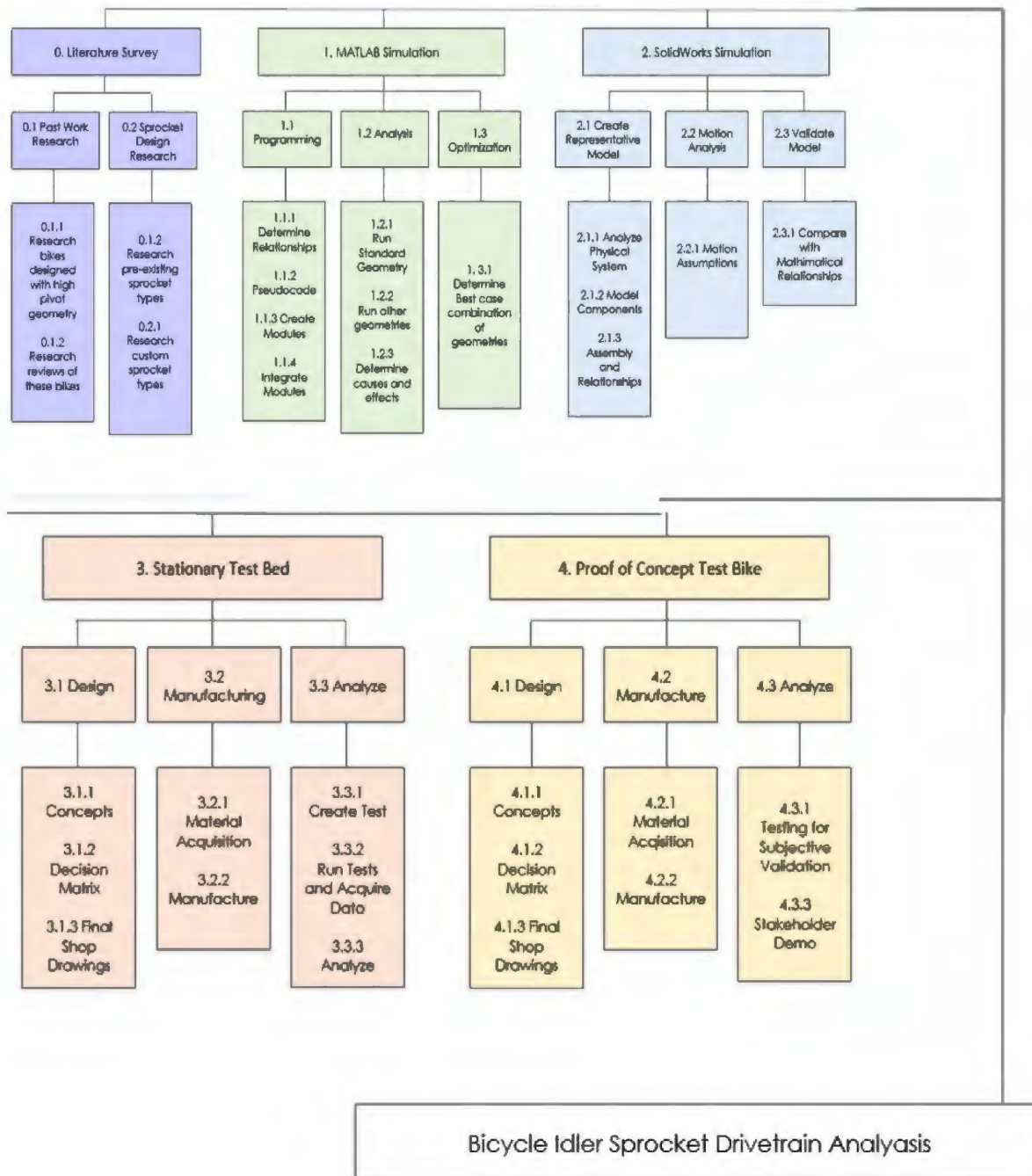
Milestone Schedule

- Complete analytical MATLAB model and SolidWorks motion analysis by Jan 1, 2019
- Design review in mid-February, 2019
- Complete testbed by the end of March, 2019
- Complete physical prototype by the end of April, 2019
- Complete stakeholder presentation in mid-May, 2019

Technical Requirement

- Using new bike for frame geometry
 - Minimum weight and therefore number of teeth
 - Minimal rumbling
 - Narrow-Wide sprocket geometry
-

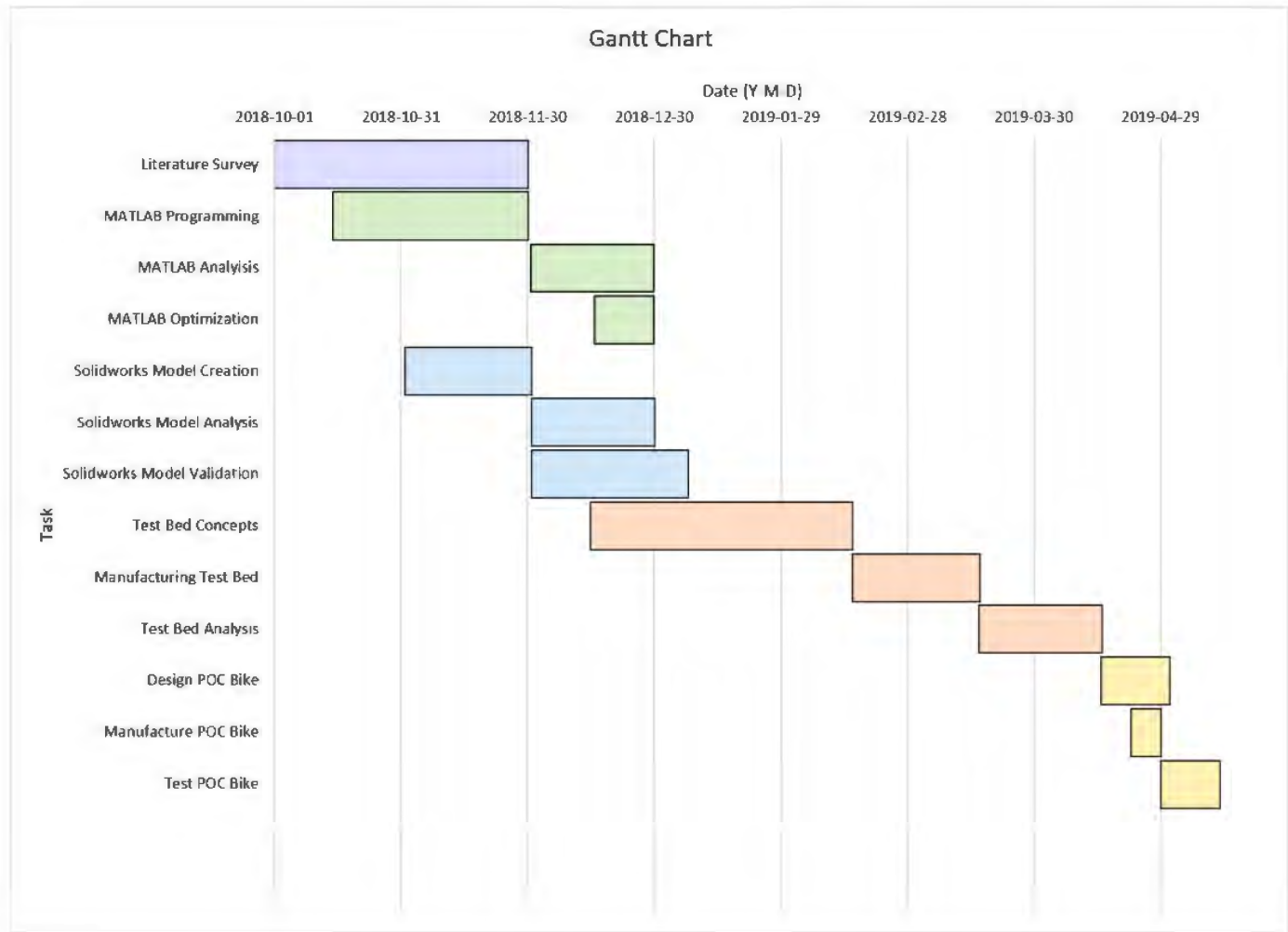
Work Breakdown Structure:



Responsibility Assignment Matrix

<i>RACI Chart</i>	Person			
Activity	Kelly	Denton	Jordan	Stephen
Literature Survey	R	A	I	I
MATLAB Code & Simulation	A	R	R	C
Solidworks Simulations	C	R	A	C
Test Bed Design	R	A	C	C
Test Bed Manufacture	R	A	R	C
Test Bed Analysis	A	C	R	C
Proof of Concept Bike Design	I	R	A	C
Sprocket Design	A	R	R	C
Sprocket Manufacture	R	I	A	C

Gantt Chart



Appendix G – Design Review Package

Executive Summary

This design review concerns the Critical Design Review (CDR) of the “Bicycle Idler Drivetrain Analysis” project. The design team hopes to display and discuss the current status of the project, and identify any weaknesses in the design, or possible optimization, including all digital and physical testing procedures.

The team has finished developing a MATLAB model of a high pivot mountain bike drivetrain. This model uses numerical methods to discretize the motion of the drivetrain and calculate the positions of sprocket teeth, the apparent acting radius of each sprocket, and therefore the gear ratio of the total system. By small movements of the idler position, and changes in idler tooth counts, the team has determined theoretical setups for increasing or decreasing vibrational amplitude.

The team is currently wrapping up the design stage and is beginning manufacturing and assembly of the physical test bench. The bench will serve as a stationary bicycle drivetrain to measure the effects of the changing gear ratio observed in the MATLAB model. This will be done using a pseudo-static method of slowly rotating the drivetrain and monitoring the gear ratio.

Current Product Development Specification (PDS)

Currently, the Project is composed of two key aspects: the mathematical model created in MATLAB, and the physical test bench.

MATLAB Model:

The model is described below using the following flow chart:

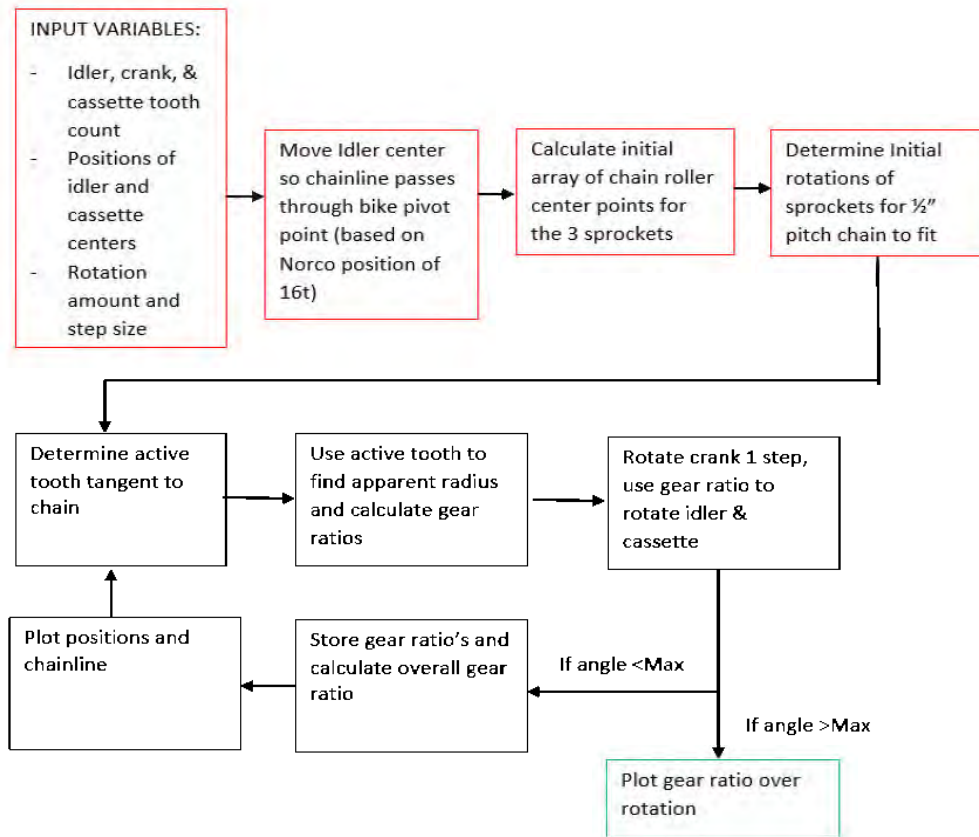


Figure G.G.1 - MATLAB Block Diagram

Physical Test Bench:

The physical test bench serves as a method to verify or dismiss the results found by the MATLAB model in a real world application directly comparable to a real high pivot bicycle. Figure 2 shows the 2D layout of the test bench, indicating how the scale measures changes in chain tension with a constant applied load.

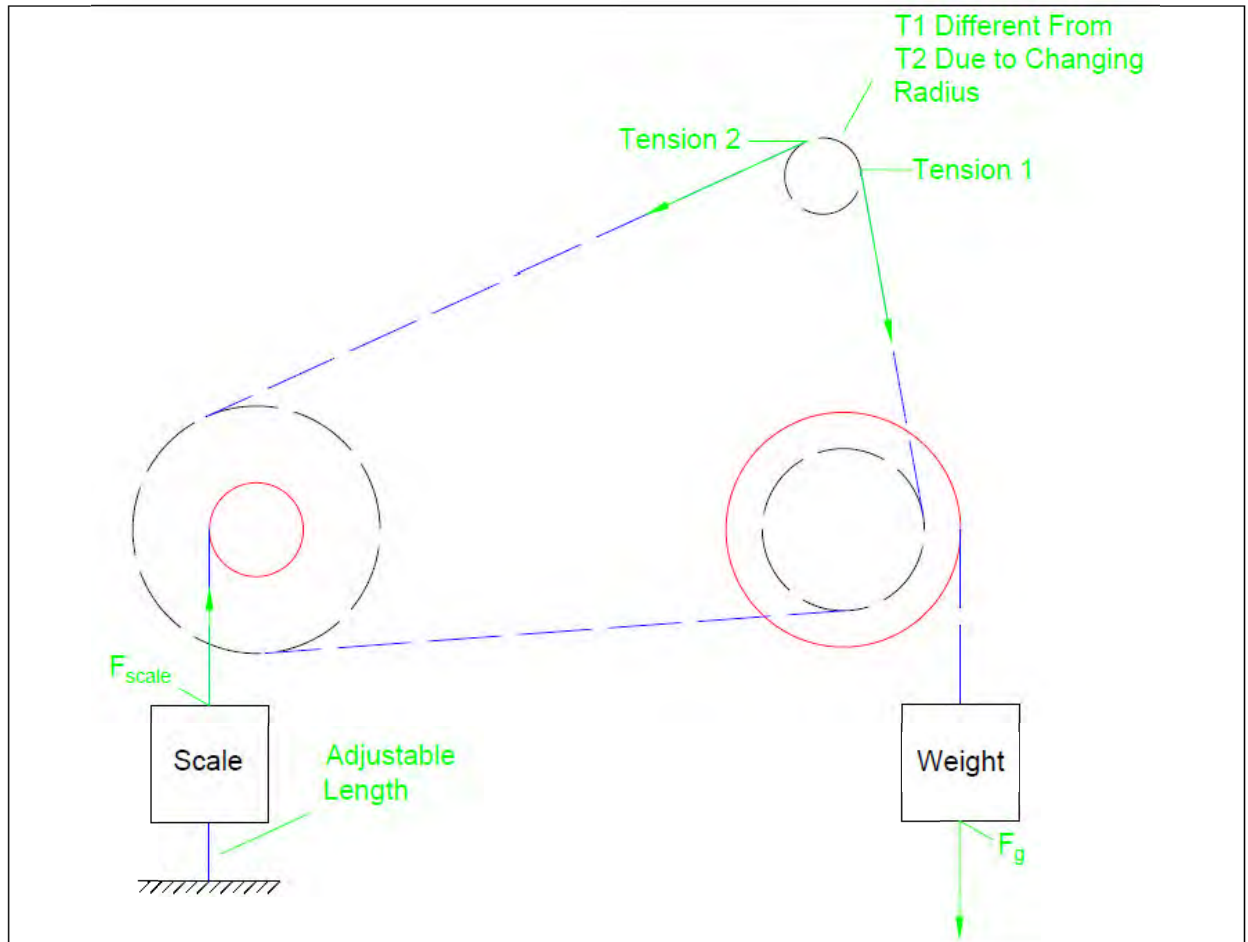


Figure G.2 - Test Bench Diagram

Engineering Data

Once the MATLAB model was confirmed to be stable and all known bugs were removed, the following results became apparent when ran with different configurations of position and tooth counts, thus determining a pattern for “good” and “bad” gear ratio profiles.

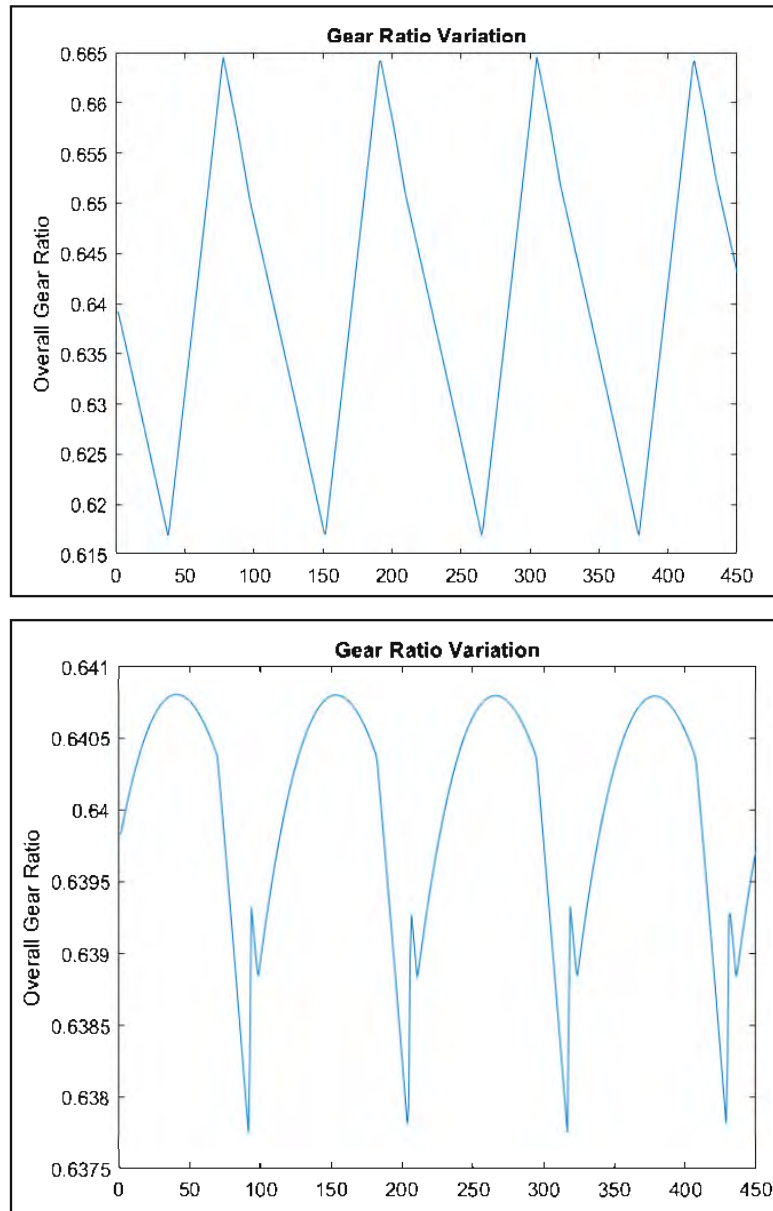


Figure G.3 - Gear Ratio Variations: “Bad” = 0.048 (top) and “Good” = 0.003 (bottom)

Every configuration simulated resulted in plots such as these, varying between the two extremes.

To further investigate the phenomenon, a sag value of 25% of a 170mm travel bike was taken as the default location and plots were created investigating the effects of changing idler tooth counts at this position (Figure 4 top) and changing x and y position from this position (Figure 4 bottom).

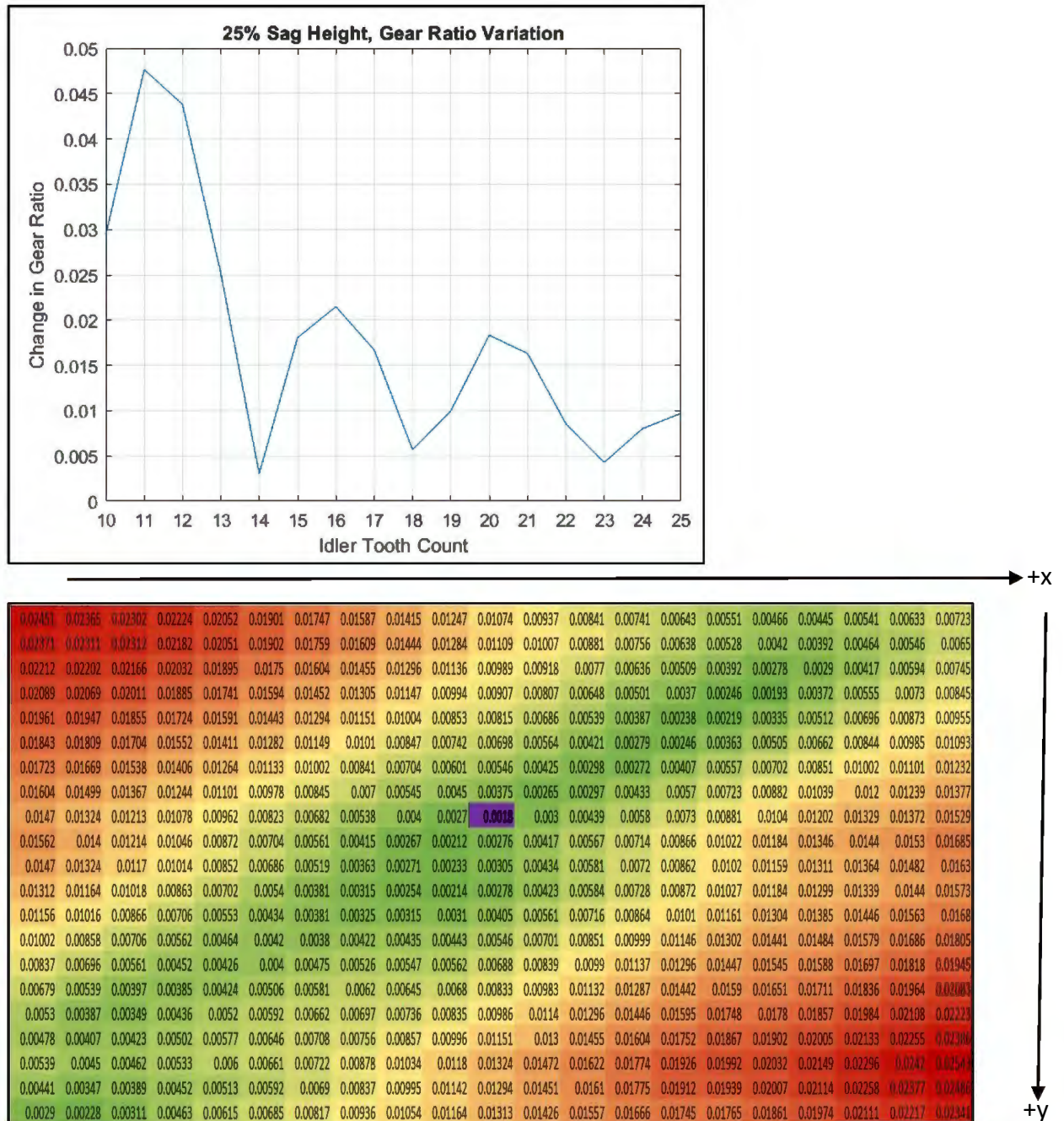


Figure G.4 - Change in Gear Ratio with tooth count change (top), and Change in Gear Ratio Over x-y Field (bottom) [red=bad, green=good, purple = best]

Several of the x-y field simulations were run, starting with different idler tooth counts, accounting for an excel file to reference the position and tooth combinations for later use in testing.

Safety Calculations:

It is important to the design team to maintain a high degree of safety during the operation of the test bed. In order to do this, the team decided on design load of 90 kg (~200 lb.). This load was chosen as an analogue to the weight of rider on the system. When combined with a 100mm radius lever, it produces approximately the same torque as 50 kg (~110lb) pushing on a traditional 175mm bicycle crank.

The following free body diagram was used to determine local forces:

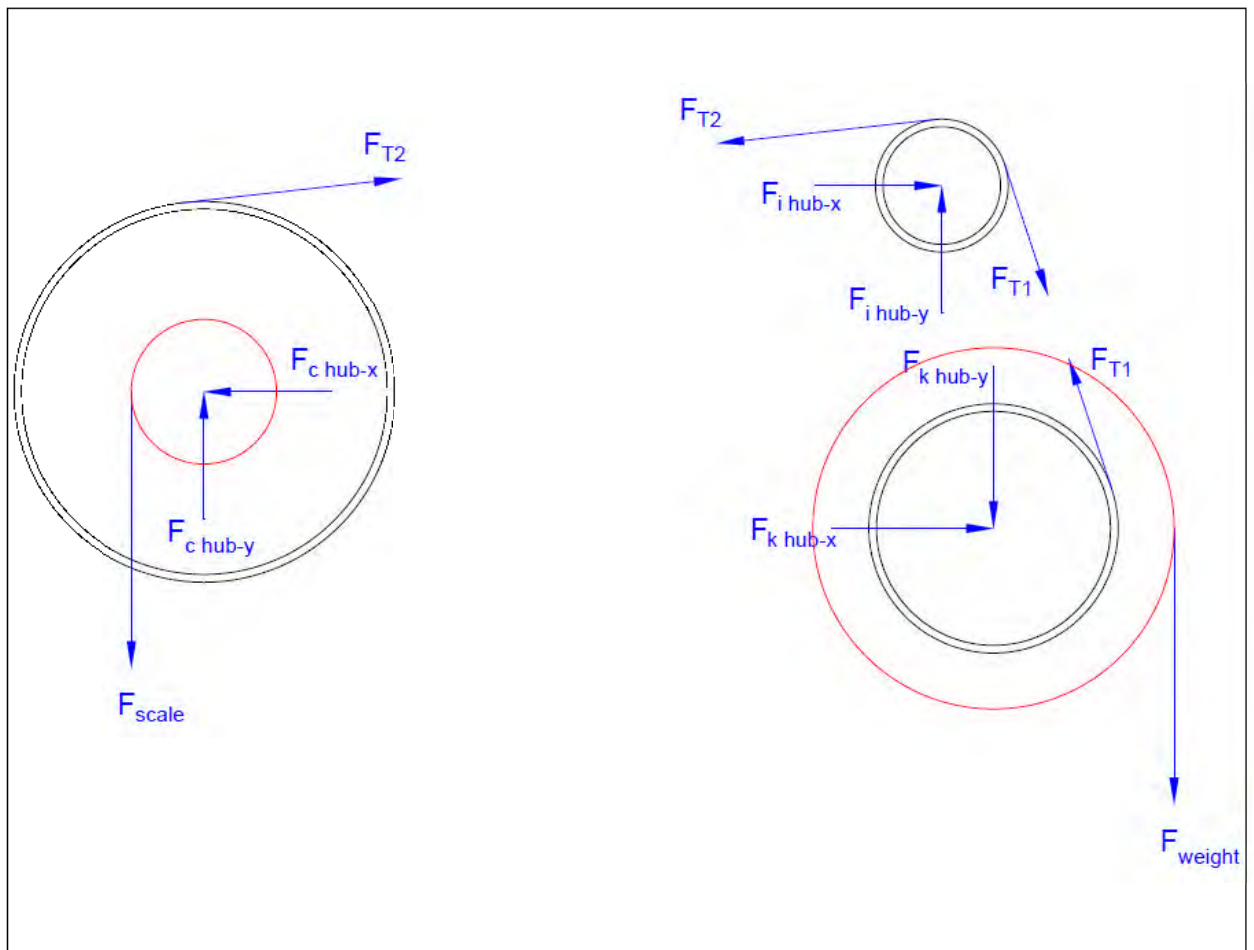


Figure G.5 - Free Body Diagram of Drivetrain

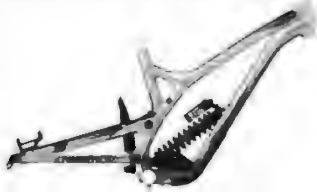


The design load is never to be exceeded in a test, and all components where applicable were sized ensuring safety factors in the following table:

Component	Load Limit	Maximum Applied Load	Safety Factor
Rope	356 kg	230 kg	1.55
Force Scale (Full Range)	270 kg	230 kg	1.17
Idler Bearing (608Z)*	2.74 kN	1.63 kN	1.68
Crank Bearings (UC204-12)	6.65 kN	1.3 kN	5.10

*Based on SFK static safety factor guidelines

Competitive Analysis of Existing Products

Currently there are few companies on the market with downhill mountain bikes utilizing high pivot suspension competing with the Norco Aurum HSP. The two main competitors in the Canadian market are the Commencal Supreme DH V4, and the GT Fury. Their main attributes are compared in the table below versus the Norco.

Bike	Commencal Supreme DH V4	GT Fury	Norco Aurum HSP
			
Price	\$2899	\$4000	\$4299
Material	Aluminum	Carbon Fiber	Carbon Fiber
Wheel Sizes	27.5 / 29	27.5 / 29	27.5 / 29
Pivot Position	High	Medium	High

Shown above it can be seen that the prices are relatively similar for the GT Fury and the Norco due to their carbon fiber construction and the Commencal is significantly less. Wheel sizes are consistent throughout with options for both 27.5” and 29” wheels. The main pivot location for the rear suspension was lower on the GT than either the Commencal or the Norco which were both quite high for regular mountain bikes. The GT seems to strike a balance between conventional and high-pivot suspension.

Online reviews were consulted for mention of any pedal feedback being felt by riders as well as real-world testing by the design team. Only one mention was found in a review by NSMB.com on the Norco, where they found the drag created by the idler was “quite noticeable”. There is the possibility that reviewers chose to omit any mention of rumbling on request from the manufacturers, or there was no noticeable rumbling.

The design team tested the two bikes themselves; a Commencal and a Norco were ridden, and slight pedal feedback could be felt in the highest gear with the worst chainline. This will be investigated on the test bench as a possible contributor to the rumbling sensation noted by Norco.

Prototypes

As discussed, the intent of this project is not to develop a product, but to further the understanding of the high-pivot suspension mountain bikes emerging on the market. As such, prototypes to be produced include a physical test bench to scientifically test the losses and vibrations in a controlled environment. The hope with this prototype is to either validate or refute the MATLAB simulation results.



Figure G.6 - Test Bench 3D Model in Solidworks

The use of 40 mm square t-slot material was chosen for the ability to create extremely fine adjustability in the x, y, and z planes allowing for tests of real world bicycle positions to be possible.

The proof of concept rig will operate by hanging a weight on the crank pulley, representing the rider force input, this tension will transfer to the chain through a chainring, over the idler sprocket and onto the cassette. From here, the hub will transfer the tension to another pulley, and will pull on a load cell. This system will slowly be let out using an eye bolt with a known thread pitch, slowly lowering the weight and “pedaling” the system. It is expected over the length of one chain pitch ($1/2''$) that the design team should see the full range of output force values as discussed in the engineering data section.

Challenges currently faced in the design and manufacture of this prototype include:

- Containing the moving parts in one safe envelope
- Possibly further improve ease of positioning and re-positioning of idler and cassette
- Improve modularity for different parts (i.e. different width hubs, different cassettes...)

Schedule Status and Projections

The progress of the project is kept on track using a Gantt chart (Figure 7). As of this time, the project is on schedule, and the project team will be ready to move onto the next phase of the project on time. The conceptual design of the physical test bed is complete, and manufacturing has begun. Due to the use of an already existing test bed frame, the team is confident that the manufacturing will be completed on time at which point data collection and analysis can begin. Time allowing, a proof of concept bike will be designed and built starting in April.

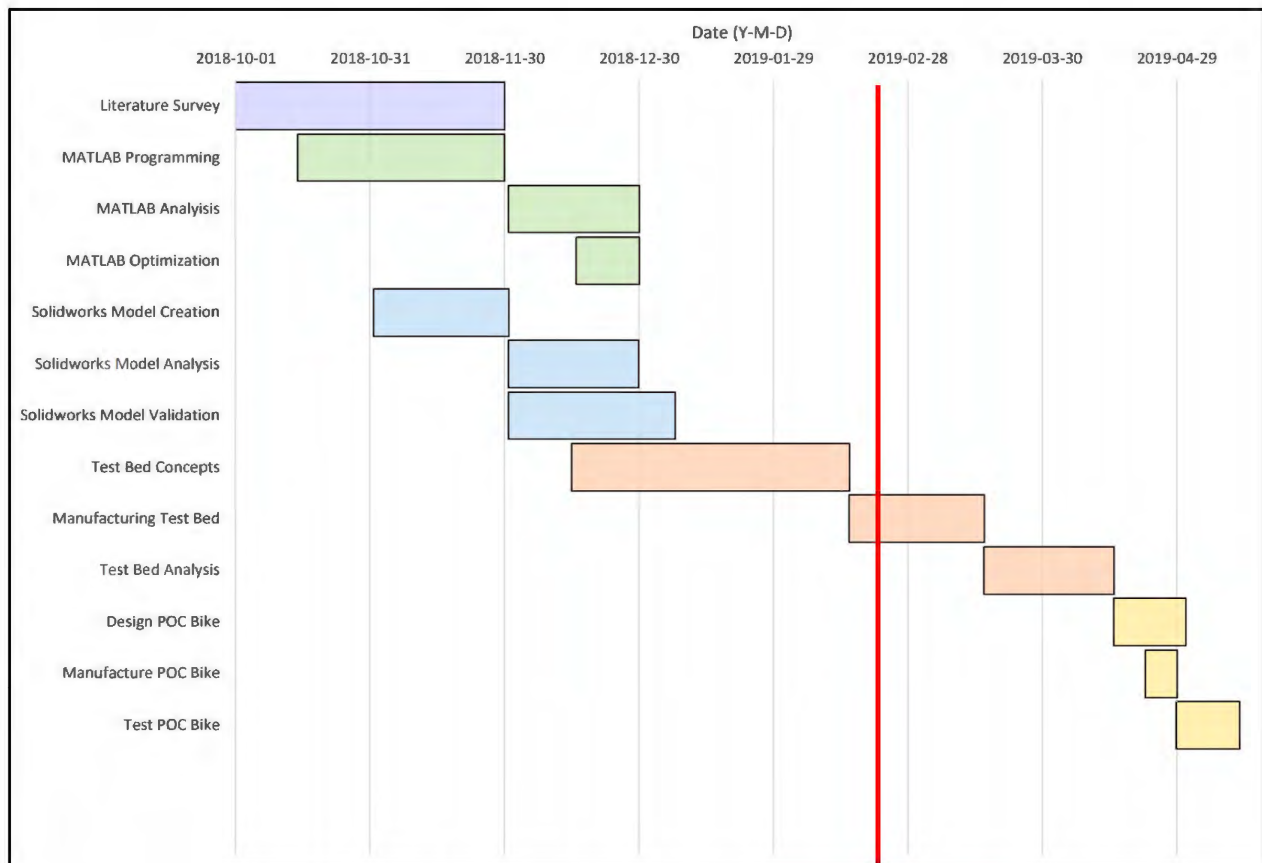


Figure G.7 - Project Gantt Chart

Project Risk Analysis

The current project risks can be broken into three categories: physical, logistic, and technical. These risks are described below as well as how the team will attempt to mitigate them.

Physical

- Free hanging weight: Due to nature of the tests, a large, free hanging weight is needed. Due to the weight being hung from a single rope, it poses a risk to anyone working on the bench as well as to the test bench itself. This risk will be mitigated by only having the weight hung during testing, as well as keeping the weight on the inside of the frame.
- Frame failure: During testing, the chain and the sprockets will be under large forces. This heightens the risk of failure in the frame where the sprockets are mounted. This is being mitigated by adding suitable safety factors during the design phase.

Logistical

- Sourcing bike parts: To finish manufacturing the test bench's drive train, bike parts including a cassette, derailleur, shifter and others. These parts are being sourced from the project's sponsor, Norco Bicycles, and the lead time is still unknown. To help deal with an excessive lead time, the team is looking for other ways to source these parts.

Technical

- Scale resolution: The rumbling of the drivetrain will be observed by measuring the change in chain tension as the chain moves over the idler. There is a risk that the scale will not have the required resolution to measure these small changes. This risk is being mitigated by designing the pulleys and using a large enough weight to make the tension changes more noticeable.

Cost Projections

Below are the tabulated costs for the various components and materials being procured for manufacturing of the test bench.

Supplier	Cost (CAD)
Raw Material (Shop Expenses)	204.78
McMaster-Carr	142.09
MiSUMi USA	35.64
Princess Auto	1.00
Total	383.51

Description of Unusual Requirements and Design Elements

There are a few unusual requirements and design elements requested by the stakeholders at Norco. Firstly, the idler tooth count must be even unless a significant reason for it not to be is found. Norco would like to keep the possibility of keeping a “narrow-wide” tooth profile currently in use. Figure 8 shows an example of this profile, which more effectively holds the chain to the idler and decreases the chances of the chain being dropped in rough terrain.

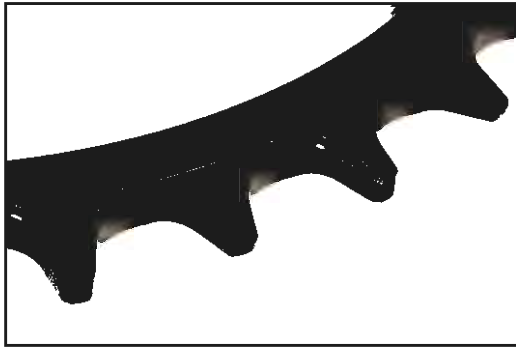


Figure G.8: Narrow wide tooth profile

Reference: https://mbaction.com/wp-content/uploads/2016/09/Rings_E13-1.jpg

A second design requirement in the event of movement or resizing of the idler, the fastening bolt for the idler must not interfere with the main pivot system. This first requirement is due to physical packaging requirements on the frame of the mountain bike.

Additionally, the pitch diameter of the idler must always pass through the center axis of the main pivot. This requirement is to help eliminate any pedal kickback caused by the upper length of the chain growing during suspension compression. If the pitch diameter did not pass through the main pivot, the upper length of the chain would grow, pulling the lower section of chain around the chain ring, rotating the pedals, and causing the rider to become unstable.

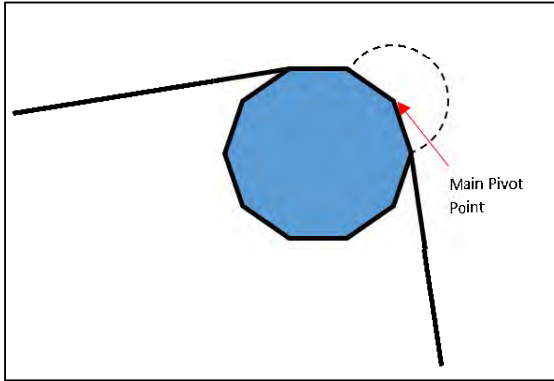


Figure G.9 - Diagram of Chainline Requirements