OpenStax Astronomy, Ch.23: WS Solutions (Apr-2021)

Solutions

1. How does a white dwarf differ from a neutron star? How does each form? What keeps each from collapsing under its own weight?

A white dwarf is an electron degenerate object, while a neutron star is a neutron degenerate object. A white dwarf has a larger radius and is much less dense than a neutron star. All white dwarfs are less than 1.4 M_{Sun} while neutron stars are between 1.4 and 3 M_{Sun}. A white dwarf forms after a star with an initial mass less than 10 M_{Sun} ends its core energy generation, while a neutron star forms from a star with an initial mass between 10 and 40 M_{Sun}. A white dwarf would form after the planetary nebula phase of a star, while a neutron star forms just before the rest of the star experiences a type II supernova detonation.

2. Describe the evolution of a star with a mass like that of the Sun, from the main-sequence phase of its evolution until it becomes a white dwarf.

After the main-sequence phase ends, the star will evolve toward the upper-right area of the H–R diagram as its core contracts and the outer layers expand. It becomes a red giant and will continue to expand its outer layers. This causes its luminosity to increase; the star's size can become more than 100 times its main-sequence radius. Eventually, the contracting core reaches a temperature of 100 million K, which leads to the explosive ignition of helium. When the star stabilizes, it will lose some of its outer layers as it becomes smaller, and moves back toward the main sequence region of the H–R diagram. The star will continue to fuse helium into carbon and oxygen, but for a time much shorter than the time on the main sequence. Eventually, the core will be depleted of helium, and the star once more evolves toward the upper-right area of the H–R diagram as the core contracts and the outer layers expand to an even greater extent. The outer layers of the star will be gradually blown out into space by the strong winds during this period. This leads to the formation of a planetary nebula out of the outer layers of the star. The remaining mass of the star in the core continues to contract and will eventually become a white dwarf.

3. Describe the evolution of a massive star (say, 20 times the mass of the Sun) up to the point at which it becomes a supernova. How does the evolution of a massive star differ from that of the Sun? Why?

A massive star will leave the main sequence once its core hydrogen is depleted. After that time, it becomes a supergiant star, and it will begin fusing a variety of heavier elements as fusion thresholds are overcome. (For example, it takes about 100 million degrees for helium to fuse into carbon.) For example, the star will first do helium fusion (where helium is the fuel), then carbon fusion, neon fusion, oxygen fusion, and silicon fusion. After the inner core becomes iron, which requires energy to fuse, the star will experience a core collapse and create a neutron star from its core. The rest of the star explodes into a supernova. The evolution differs from a solar-mass star in several ways. First, the fusion history is different: A massive star can fuse more elements and create more fusion products. Second, the lifespan for each star is quite different, with a solar-mass star taking a much longer time for each stage of its evolution compared to the massive star. Third, the method by which each star loses mass is different, with a solar-mass star losing mass via the planetary nebula phase, while a massive star undergoes a supernova event. Finally, the final state of each star is different, with a low-mass star becoming a white dwarf, and a massive star becoming a neutron star. The main factor that determines the fate of each star and the path they will take to get to the fate is their initial mass on the main sequence.

4. How do the two types of SN discussed in this chapter differ? What kind of star gives rise to each type?

Type Ia supernovae are produced by white dwarf stars in a binary star system that have exceeded their Chandrasekhar limit when the companion star dumps a lot of material onto them. Type II supernovae are produced by massive stars whose cores collapse following the exhaustion of their fusion processes. Type Ia supernovae are more luminous than type II supernovae and have a more consistent value for maximum brightness. Type Ia supernova are also visible in all types of galaxies, while type II are not observed in all galaxies.

5. A star begins its life with a mass of 5 M_{Sun} but ends its life as a white dwarf with a mass of 0.8 M_{Sun}. List the stages in the star's life during which it most likely lost some of the mass it started with. How did mass loss occur in each stage?

A very small fraction of mass is lost while on the main sequence through the conversion of hydrogen into helium, and in later fusion cycles (where mass is converted into energy), as well as through stellar winds. Mass is lost during the red giant phase, as well as during the time the outer layers lift off as a planetary nebula.

6. If the formation of a neutron star leads to a supernova explosion, explain why only three of the hundreds of known pulsars are found in supernova remnants.

Pulsars can remain visible through their radio emission much longer than the material from a supernova explosion stays visible. Also pulsars can be "kicked" from the site of the supernova through a process that causes them to move away from the remnant at very high velocities.

7. How can the Crab Nebula shine with the energy of something like 100,000 Suns when the star that formed the nebula exploded almost 1000 years ago? Who "pays the bills" for much of the radiation we see coming from the nebula?

The pulsar is providing the energy through the emission particles from its magnetic poles. These high velocity particles emit a range of energy, and they interact with and energize the material that was expelled during the supernova explosion. This energy source is ultimately powered by the rotation of the pulsar in the center of the Crab Nebula. As it emits particles, the pulsar loses energy and spins slower.

8. How is a nova different from a type Ia supernova? How does it differ from a type II supernova?

A nova is a smaller energy explosion on the surface of a white dwarf in a close binary system, where fresh material from a donor star is deposited on the surface of the white dwarf until it ignites. A type Ia supernova has a similar configuration, but in this case, the material deposited on the surface of the white dwarf is sufficient to push the white dwarf past the Chandresekhar limit. Once that happens, the white dwarf will collapse and then explode into a type Ia supernova. A type II supernova does not involve a white dwarf but instead requires a massive star to reach the end of its ability to generate energy in its core. This results in a collapse of the core and an explosion into a type II supernova.

9. What observations from SN 1987A helped confirm theories about supernovae?

The light variation from the supernova changed due to the influence of radioactive elements decaying. This included the decay of nickel-56 and cobalt-56. As these elements decayed, their gamma rays interacted with the material from the explosion, causing the energy output to change. Since these elements decay very quickly, they must have been created by the supernova explosion itself; they would not have survived if they had been created during earlier stages of the star's life. Neutrinos were also observed from the supernova on the day before the light from the supernova was observed. If enough neutrinos were created so that a good number reached Earth, 160,000 light-years away, and a few could interact with our detectors, this shows that a lot of neutrinos must have been made by the events connected with the supernova. This confirms our idea that in the formation of the neutron star, before the explosion, a large number of neutrinos were created in the process of joining electrons and protons to make neutrons.

10. Describe the evolution of a white dwarf over time, in particular how the luminosity, temperature, and radius change.

The only feature that doesn't change appreciably is the radius. The temperature of a white dwarf will decrease as it radiates away its energy slowly. The change in temperature is the only factor to change the luminosity, and that results in a decrease of luminosity over time. Eventually, white dwarfs become black dwarfs, as only infrared radiation is given off.

11. Describe the evolution of a pulsar over time, in particular how the rotation and pulse signal changes over time.

As beams of particles and their associated energy are given off, the pulsar will lose energy slowly, which will decrease the rate of its rotation. The frequency of pulses would therefore decrease, so that fewer pulses are observed in a given time span. The strength of the pulse signal will also decrease so the pulses will become fainter. Eventually, the pulsar should rotate so slowly and have such a low emission of radiation that it would no longer be observable.

12. How would a white dwarf that formed from a star that had an initial mass of 1 M_{Sun} be different from a white dwarf that formed from a star that had an initial mass of 9 M_{Sun}?

The lower initial mass white dwarf would be composed primarily of carbon and oxygen, while the higher initial mass star would form a white dwarf with higher mass elements, such as oxygen, neon, and magnesium.

13. What do astronomers think are the causes of longer-duration gamma-ray bursts and shorter-duration gamma-ray bursts?

Long-duration bursts (lasting more than a few seconds) come from massive stars with their outer hydrogen layers missing that explode as supernovae. Short duration bursts are believed to be mergers of stellar corpses (neutron stars or black holes).

14. How did astronomers finally solve the mystery of what gamma-ray bursts were? What instruments were required to find the solution?

The problem was that early gamma-ray telescopes did not allow astronomers to pinpoint the location of the bursts very precisely—only within a box on the sky that could include a lot of different objects. What it took was finding the "afterglow" of bursts in other wavelengths (X-rays, visible light, radio). For those other wavelengths, telescopes could measure the location of the afterglow much more precisely and find what the objects were in which the bursts happened. Satellites like *Swift* have telescopes in several bands of the electromagnetic spectrum; once a gamma-ray burst location is identified, the satellite swivels automatically to point X-ray and other telescopes at the location.

15. Arrange the following stars in order of their evolution:

A. A star with no nuclear reactions going on in the core, which is made primarily of carbon and oxygen.

B. A star of uniform composition from center to surface; it contains hydrogen but has no nuclear reactions going on in the core.

C. A star that is fusing hydrogen to form helium in its core.

D. A star that is fusing helium to carbon in the core and hydrogen to helium in a shell around the core.

E. A star that has no nuclear reactions going on in the core but is fusing hydrogen to form helium in a shell around the core.

The order, youngest to oldest, is as follows: B, which is a protostar contracting toward the main sequence; C, which is a main-sequence star undergoing the first stage of nuclear fusion; E, which is a star that has exhausted hydrogen in its core but has contracted enough to increase its temperature to the point where hydrogen fusion can take place around the core; D, which is a star that has contracted and increased its core temperature to the point where helium fusion is possible; and A, which is a white dwarf that has exhausted its hydrogen and helium and no longer can produce fusion reactions.

16. Suppose no stars more massive than about 2 M_{Sun} had ever formed. Would life as we know it have been able to develop? Why or why not?

Stars with masses less than two times the mass of the Sun can produce elements only up to carbon and oxygen. More massive elements are not produced, and some of these more massive elements (phosphorus, calcium, silicon, iron) are essential for the forms of life found on Earth.

17. Would you be more likely to observe a type II supernova (the explosion of a massive star) in a globular cluster or in an open cluster? Why?

A type II supernova is thought to be produced when a massive star explodes. All the massive stars in globular clusters (which formed early in the history of our Galaxy) completed their evolution long ago, and so we would not expect to see a type II supernova in a globular cluster today. One might occur in a very young open cluster, where very massive stars might just be finishing their lives. (This answer ignores the recent work that indicates that in globular clusters, individual lower-mass stars may later collide and become a high-mass star in rare instances. Since this is not discussed in the text, students are not likely to discuss this.)]

18. If most stars become white dwarfs at the ends of their lives and the formation of white dwarfs is accompanied by the production of a planetary nebula, why are there more white dwarfs than planetary nebulae in the Galaxy?

A white dwarf is visible for a billion years or more before it cools off and its radiation becomes so feeble as to be undetectable. After a time on the order of 10,000 years or so, the gas shell that is ejected in the planetary nebula phase expands and thins out to such an extent that it becomes unobservable. Therefore, there are many more white dwarfs than planetary nebulae.

19. If a 3 and 8 M_{Sun} star formed together in a binary system, which star would:

A. Evolve off the main sequence first?

B. Form a carbon- and oxygen-rich white dwarf?

C. Be the location for a nova explosion?

A. 8 M_{Sun}; B. 3 M_{Sun}; C. 8 M_{Sun}.

20. You have discovered two star clusters. The first cluster contains mainly main-sequence stars, along with some red giant stars and a few white dwarfs. The second cluster also contains mainly main-sequence stars, along with some red giant stars, and a few neutron stars—but no white dwarf stars. What are the relative ages of the clusters? How did you determine your answer?

The first cluster is older than the second one. Stars of all mass become giants as part of their evolution. The lack of white dwarfs in the second cluster indicates that only the most massive stars have evolved to the end state of their evolution, which happens very quickly for high-mass stars. The lower-mass stars in the second cluster have not yet evolved to the white dwarf stage. The first cluster is old enough for the lower mass stars to reach the white dwarf stage.

21. How would the spectra of a type II supernova be different from a type Ia supernova? Hint: Consider the characteristics of the objects that are their source.

A type II supernova is formed from the collapse of a massive star, which, although it has made heavier elements in its core, is still mainly composed of hydrogen and helium. These should be visible in the spectrum, along with the other elements produced in the supernova. However, the amount of hydrogen and helium is still significantly larger than the other elements. A type Ia supernova is formed from a white dwarf star, which contains elements other than hydrogen, such as carbon, oxygen, neon, and magnesium. The spectrum of a type Ia supernova would show spectral features associated with elements other than hydrogen. (Even if the other star is dumping hydrogen onto the white dwarf, the tremendous compression and heating, and then the explosion, will convert that hydrogen to heavier elements.)

22. One way to calculate the radius of a star is to use its luminosity and temperature and assume that the star radiates approximately like a blackbody. Astronomers have measured the characteristics of central stars of planetary nebulae and have found that a typical central star is 16 times as luminous and 20 times as hot (about 110,000 K) as the Sun. Find the radius in terms of the Sun's. How does this radius compare with that of a typical white dwarf?

The total energy put out by a star is given by $L = 4\pi R^2 T^4$. To find the ratio of the size of the central star of the planetary nebula to the size of the Sun, we need to do the ratio:

$$\left[\frac{R_{Sun}}{R_{star}}\right]^2 = \frac{\left(\frac{L_{Sun}}{L_{star}}\right)}{\left[\frac{T^4_{Sun}}{T^4_{star}}\right]} = \frac{\left(\frac{1}{16}\right)}{\left(\frac{1}{20}\right)^4} = 10{,}000.$$

Therefore, R_{Sun} is 100 times larger than R_{star}. Since the diameter of the Sun is about 100 times larger than the diameter of Earth, this means that the central star has about the diameter of Earth, which is typical for a white dwarf.

23. According to a model described in the text, a neutron star has a radius of about 10 km. Assume that the pulses occur once per rotation. According to Einstein's theory of relatively, nothing can move faster than the speed of light. Check to make sure that this pulsar model does not violate relativity. Calculate the rotation speed of the Crab Nebula pulsar at its equator, given its period of 0.033 s. (Remember that distance equals velocity × time and that the circumference of a circle is given by $2\pi R$).

For the Crab pulsar, we have $v_{\text{rotation}} = \frac{2\pi R}{t} = \frac{2\pi \times 10}{0.033} = 1903 \text{ km/s}$, which is well under the speed of light.

24. Before the star that became SN 1987A exploded, it evolved from a red supergiant to a blue supergiant while remaining at the same luminosity. As a red supergiant, its surface temperature would have been approximately 4000 K, while as a blue supergiant, its surface temperature was 16,000 K. How much did the radius change as it evolved from a red to a blue supergiant?

Since the luminosity remains the same throughout, the ratio of the temperatures can be used to determine the ratio of the radii using the luminosity-radius-temperature relationship. Setting up the relationship for the red and blue supergiant gives the following:

$$\frac{L_{\text{red}}}{L_{\text{blue}}} = \frac{R_{\text{red}}^2 T_{\text{red}}^4}{R_{\text{blue}}^2 T_{\text{blue}}^4}$$

$$1 = \frac{R_{\text{red}}^2 4000^4}{R_{\text{blue}}^2 16000^4} = \frac{R_{\text{red}}^2}{R_{\text{blue}}^2} 0.0039$$

$$\frac{1}{0.0039} = 260 = \frac{R_{\text{red}}^2}{R_{\text{blue}}^2}$$

$$\sqrt{260} = 16 = \frac{R_{\text{red}}}{R_{\text{blue}}}$$

As a red supergiant, the star had a radius that was 16 times larger than when it became a blue supergiant.