OpenStax Astronomy, Ch.18: WS Solutions (Apr-2021)

Solutions

1. How does the mass of the Sun compare with that of other stars in our local neighborhood?

> The Sun is more massive than the majority of stars in our neighborhood. Only a few other stars are more massive, whereas the vast majority are lower-mass stars.

2. Name and describe the three types of binary systems.

> The three types of binary systems are spectroscopic, visual, and eclipsing. A spectroscopic binary star is a binary star in which the components are not seen separately, but whose binary nature is indicated by periodic variations in radial velocity (changes in the Doppler shift of the spectral lines), indicating orbital motion. A visual binary is a binary star in which the two components are telescopically resolved (can be seen individually). An eclipsing binary star is a binary star in which the plane of revolution of the two stars is nearly edge-on to our line of sight, so that periodically, one star blocks the light of the other by passing in front of it.

3. Describe two ways of determining the diameter of a star.

> In one method, the time for an object like the Moon to pass in front of a star can be measured to determine the diameter of a star. Since we know the speed of the Moon in its orbit, we can calculate the size of the star. For an eclipsing binary star, the time for one star to pass in front of another is dependent upon the relative diameters of each star. When the eclipses are aligned in such a way that they eclipse each other, we can measure the time for each star to eclipse the other. We can measure the speed of the stars from the Doppler shift in the spectrum. From knowing the time of eclipse and the speed, the size of each star can be determined.

4. What are the largest- and smallest-known values of the mass, luminosity, surface temperature, and diameter of stars (roughly)?

> Mass ranges from more than 100 times the Sun’s mass (up to 250 times the Sun’s mass) down to 1/12 the Sun’s mass. Luminosity ranges from a million times the Sun’s luminosity down to 1/10,000 of the Sun’s. Surface temperature ranges from nearly 40,000 K down to 2700 K. Diameter ranges from 1000 times the Sun’s diameter down to 1/10 the Sun’s diameter.

5. Sketch an H–R diagram. Label the axes. Show where cool supergiants, white dwarfs, the Sun, and main-sequence stars are found.

> Results are shown in Figure 18.14 H–R Diagram for a Selected Sample of Stars and Figure 18.15 Schematic H–R Diagram for Many Stars.

6. Describe what a typical star in the Galaxy would be like compared to the Sun.

> Cool, faint, low-mass stars located on the lower part of the main sequence are the most common, and therefore the most typical stars.

7. How do we distinguish stars from brown dwarfs? How do we distinguish brown dwarfs from planets?

> Stars have mass greater than 1/12th of the Sun’s mass; brown dwarfs generally have between 1/100th and 1/12th the mass of our Sun; planets have masses less than that. (In future chapters, a different criterion—the type of fusion each one experiences or doesn’t experience—will be added to this list.)

8. Describe how the mass, luminosity, surface temperature, and radius of main-sequence stars change in value going from the “bottom” to the “top” of the main sequence.

At the bottom, the mass, luminosity, surface temperature, and radius are all at their lowest values. As you head to the top of the main sequence, the values all increase and are at a maximum at the top. The values that change the most are luminosity and temperature. Radius has the least amount of change in value. This is based upon the values in Table 18.3 Characteristics of Main-Sequence Stars.

9. Why do most known visual binaries have relatively long periods and most spectroscopic binaries have relatively short periods?

Visual binaries must be rather well separated to be detected as such. Thus, they generally have large semimajor axes, and by Kepler's third law, long periods and low orbital speeds. Spectroscopic binaries must have rather high orbital velocities so that the effect of this motion is clearly identifiable in the spectrum. Hence, they tend to have short periods. High orbital velocity occurs with long periods and large major axes only if the stars are very massive.

10. Review this spectral data for five stars.

Star	Spectrum
1	G, main sequence
2	K, giant
3	K, main sequence
4	O, main sequence
5	M, main sequence

Which is the hottest? Coolest? Most luminous? Least luminous? In each case, give your reasoning.

Hottest: Star 4, spectral type determines relative temperature, O is the hottest in the list. Coolest: Star 5, spectral type determines relative temperature, M is the coolest in the list. Most luminous: Star 4, with the highest temperature, and upper main sequence location places it at the upper-left corner of the H–R diagram. Least luminous: Star 5, with the lowest temperature and lowest main-sequence location, which places it in the lower-right corner of the H–R diagram.

11. Which changes by the largest factor along the main sequence from spectral types O to M—mass or luminosity?

According to the text, the mass varies from only about 100 M_{Sun} to about 0.08 M_{Sun}, or by about a factor of 1000 along the main sequence, whereas the luminosity varies from 10^6 for the most luminous stars to less than 0.001 or a factor of more than 10^9 for the least luminous. The text says only that the radii of main sequence stars are fairly similar to that of the Sun, which implies that the radii change by a smaller factor than the luminosity. The validity of this conclusion can be shown by noting that the luminosity of a star is proportional to R^2T^4, but this is beyond what is covered in this text.

12. Suppose you want to search for brown dwarfs using a space telescope. Will you design your telescope to detect light in the ultraviolet or the infrared part of the spectrum? Why?

Very low-mass stars or brown dwarfs are relatively cool, with temperatures of only about 2000 K. Such stars emit most of their light in the infrared and practically none in the ultraviolet.

13. An astronomer discovers a type-M star with a large luminosity. How is this possible? What kind of star is it?

Since M stars are cool and emit very little energy per unit area (say per square meter), the only way that an M star can have a high luminosity is if it is very large (i.e., has a lot of square meters of surface area). This star is either a giant or a supergiant.

14. Approximately 6000 stars are bright enough to be seen without a telescope. Are any of these white dwarfs? Use the information given in this chapter to explain your reasoning.

None of the stars visible to the unaided eye are white dwarfs. White dwarfs are hot but not very luminous since their surface area is so small. Just as we need telescopes to see low-luminosity, main-sequence stars, we need telescopes to see white dwarfs. (For the instructor, to be visible to the naked eye, a typical white dwarf would have to be at a distance of 3.26×10^{-3} light-years. The nearest star, Alpha Centauri, is at a distance of 4.3 light-years.)

15. If you were to compare three stars with the same surface temperature, with one star being a giant, another a supergiant, and the third a main-sequence star, how would their radii compare to one another?

The main-sequence star would have the smallest radius, whereas the giant would have a larger radius, and the supergiant would have the largest radius. This is based upon the relationship of radius, temperature, and luminosity.

16. It is possible that stars as much as 200 times the Sun's mass or more exist. What is the luminosity of such a star based upon the mass-luminosity relation?

Using the formula, you'd get

$$\frac{L}{L_{\text{Sun}}} = \left(\frac{M}{M_{\text{Sun}}}\right)^4 = (200)^4 = (2 \times 10^2)^4 = 1.6 \times 10^1 \times 10^8 = 1.6 \times 10^9$$

times the Sun's luminosity.

17. The lowest mass for a true star is 1/12 the mass of the Sun. What is the luminosity of such a star based upon the mass-luminosity relationship?

Using the formula, you'd get

$$\frac{L}{L_{\text{Sun}}} = \left(\frac{M}{M_{\text{Sun}}}\right)^4 = \left(\frac{1}{12}\right)^4 = (0.083)^4 = 4.7 \times 10^{-5} \text{ times the Sun's luminosity.}$$

18. How much would you weigh if you were suddenly transported to the white dwarf Sirius B? You may use your own weight (or if don't want to own up to what it is, assume you weigh 70 kg or 150 lb). In this case, assume that the companion to Sirius has a mass equal to that of the Sun and a radius equal to that of Earth. Remember Newton's law of gravity: $F = GM_1M_2/R^2$ and that your weight is proportional to the force that you feel. What kind of star should you travel to if you want to *lose* weight (and not gain it)?

The force of gravity that you feel on the surface of Earth is what you sense as weight. That force is necessary to compare to the force you'd feel on Sirius' companion:

$$F_{\text{Earth}} = \frac{GM_{\text{Earth}}M_{\text{You}}}{(R_{\text{Earth}})^2} = \frac{(6.672 \times 10^{11}\ \text{Nm}^2/\text{kg}^2)(5.977 \times 10^{24}\ \text{kg})(70\ \text{kg})}{(6.378 \times 10^6\ \text{m})^2} = 690\ \text{N}.$$

For Sirius B,

the only change to the above relation is the change in the mass of the object, since we are assuming its radius is the same as Earth, and your mass doesn't change:

$$F_{\text{Companion}} = \frac{GM_{\text{Companion}}M_{\text{you}}}{\left(R_{\text{Earth}}\right)^2} = \frac{(6.672 \times 10^{11}\ \text{Nm}^2/\text{kg}^2)(1.989 \times 10^{30}\ \text{kg})(70\ \text{kg})}{(6.378 \times 10^6\ \text{m})^2} = 2.3 \times 10^8\ \text{N}.$$

This value is approximately 330,000 times greater than the force you feel on Earth, so you would basically feel like you weigh 330,000 times greater than you do now. If you want to go to a star that would allow you to feel as if you weigh less, you'd want to go to a place that has either a lower mass or a greater radius—both of these would result in a lower gravitational force. Since most of the stars in the Sun's neighborhood are low-mass M-dwarfs, those would be easiest to find.

19. The star Betelgeuse has a temperature of 3400 K and a luminosity of 13,200 L_{Sun}. Calculate the radius of Betelgeuse relative to the Sun.

The total luminosity of a star is given by the total energy emitted per square meter times the total surface area of the star. According to the information given in the problem, we thus have $L_{\text{star}} = 4\pi R_{\text{star}}{}^2\sigma T^4$. The radius of the star squared is then given by $R_{\text{star}}{}^2 = L_{\text{star}}/(4\pi\sigma T_{\text{star}}{}^4)$. A similar equation applies to the Sun. If we then take the ratio of the two equations, and use the temperature of the Sun of 5800 K, $(R_{\text{star}}/R_{\text{Sun}})^2 = (L_{\text{star}}/L_{\text{Sun}})(T_{\text{Sun}}{}^4/T_{\text{star}}{}^4) = (10^5)(5800/3000)^4 = 1.4 \times 10^6$, $R_{\text{star}}/R_{\text{Sun}} = 1200$. The radius of Betelgeuse is 1200 times larger than the radius of the Sun and would stretch beyond the orbit of Jupiter.

20. Confirm that the angular diameter of the Sun of 1/2° corresponds to a linear diameter of 1.39 million km. Use the average distance of the Sun and Earth to derive the answer. (Hint: This can be solved using a trigonometric function.)

To solve using a trigonometric function, the angle centered on Earth would be 1/4°, the distance opposite of this angle would correspond to the radius of the Sun (rather than the diameter), and the distance from Earth to the center of the Sun's disk would be 1 AU. The function to use would be tangent $\left(\frac{1}{4}\right) = \frac{\text{radius of the Sun}}{1\ \text{AU}}$. Radius = 1.496×10^8 km × tan(0.25) = 6.5×10^5 km. This gives a diameter of 1.3×10^6 km, similar to the value quoted in the text.